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AN APPLICATION OF LINEAR PROGRAMMING
TO RECRUIT TRANSPORTATION

C. O. ANDERSON
P. van R. SCHOEFFEL

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AN APPLICATION
OF LINEAR PROGRAMMING TO
RECRUIT TRANSPORTATION

* * * * *

C. O. ANDERSON

and

P. VAN R. SCHOEFFEL

AN APPLICATION
OF LINEAR PROGRAMMING TO
RECRUIT TRANSPORTATION

by

C. O. ANDERSON

Lieutenant Commander, United States Navy

and

P. van R. SCHOEFFEL

Lieutenant Commander, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The problem of transporting recruits from Navy Recruiting Stations to Recruit Training Centers in such a way as to minimize total cost is subjected to solution by standard linear programming methods. A sensitivity analysis of the variation of total cost for different modes of transportation, as influenced by the variation of the proportion of recruits going to each of the two Recruit Training Centers, is carried out. A probabilistic model of the occurrence of a given number of enlistments, up to a fixed quota, at each Recruiting Station is formulated and a simulation of the transportation problem arising is carried out. A rationale is developed for the derivation of a realistic estimator for planning, and some of the difficulties of determining a best estimator are discussed. A computer program for approximating the expected loss for any estimator is presented. Some other types of approach to the problem are indicated.

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1. Introduction.

In today's Navy considerable emphasis is being put on cost reductions of all forms. One important example of an area in which significant cost reductions might be made is that of the transportation of recruits from Navy Recruiting Stations (RS's) to Recruit Training Centers (RTC's). The large number of such "shipments" made indicate that close study of the costs are needed. Accordingly, the Bureau of Naval Personnel, in their letter PERS B-61 KAD of 29 May 1963, requested investigation of methods to reduce these costs. This paper is presented as an illustration of a practical application of the basic linear programming method of solving transportation problems as applied to naval recruits from the time of their enlistment until their arrival at the appropriate RTC.

There are thirty-nine main Naval Recruiting Stations and nineteen class "A" substations, a total of fifty-eight recruit sources. Each RS is assigned, as a quota, a certain percentage of total number of recruits to be included by the Navy Recruiting System for the year. To receive the recruits there are two RTC's each of which is assigned to receive half the total number of recruits enlisted. Recruits can, in principle, be sent from any RS to either RTC by any of three modes of transportation: air, rail, or bus, all of which cost different amounts. In addition, there are differences in cost which are a result of obtaining group rates. In order to analyze the cost of transportation more than superficially, it is necessary to include costs other than those of tickets, e.g., meals, miscellaneous (limousine service, etc.) and pay of recruits during transit. The desired solution of the problem should give the optimum number of recruits to be sent to each RTC from

each RS and should determine the modes by which they should be sent. In addition, since fare rates, cost of living, pay rates, RS quotas, and RTC quotas may all vary, it is necessary to determine the effects of all of these changes on the optimum transportation allocations. Determinations of these effects are known as sensitivity analyses.

This problem, with slight modifications, may be formulated as the linear programming transportation problem. This formulation of the problem is that of the shipment of a homogeneous product from a stated number of sources to a stated (possibly different) number of receiving points (destinations); the amount to be shipped from each source and the amount to be shipped to each destination is predetermined; however, the amount going from a specific source to a specific destination is not. The cost of shipment of one item from any source to any destination is a necessary input. The solution determines the amounts to be shipped from each source to each destination in such a manner as to minimize total cost, and gives the minimum total cost.

In this paper we have made use of a previously written FORTRAN program of the simplex algorithm to solve (on a CDC 1604 computer) the basic transportation problem as posed by the Bureau of Naval Personnel. In addition we have investigated the changes in the solution introduced by varying certain of the constraints (e.g., the restriction that 50% of the men go to each Recruit Training Center). Further, an attempt has been made to come to grips with the more realistic problem in which the number of recruits desired is specified, but the number which will actually enlist can be described only probabilistically.

Since some of the data were unavailable at the time this paper was written, those values were supplied by educated guess. Further,

the actual number of men in the quotas were not known; the percentages of the total were used instead. Thus, this paper is a demonstration of a method which becomes a true solution (within the limits of its assumptions) only when the true values are supplied.

2. Problem Formulation.

In this (classical) statement of the linear programming transportation problem the following information is necessary as input:

It is desired to ship a homogeneous product from each of a given number of destinations in such a manner that each destination receives an exact desired amount (which can be different for each destination). Further, this shipment is to be carried out in such a manner that each origin sends an exact desired amount which can be different for each origin. The total amount that is desired to send is equal to the total amount that is desired to receive. No shipment takes place from destinations to origins. The problem is to be solved in such a way as to minimize the total cost of all this transportation. In other problems some of the above equalities may be inequalities, but in our problem equalities are adequate.

To solve our problem realistically, account must be taken of the fact that seldom will money savings be the only governing criterion in any problem involving people. Hence, it will be necessary to provide solutions to the transportation problem under consideration, which for example, might be to prevent trip length from being excessive. Suppose a decision were made (for reasons of morale, say) that no recruit should spend more than thirty-six hours in travel. Obviously for some RS's the cheapest mode of transportation, which is also the slowest, would no longer be usable, and some costs would be changed. This would raise the total cost of transporting recruits and might well change the pattern of shipments, i.e., how many men are sent between which sources and destinations, for minimizing total cost using the revised costs. Another variation introduced into the problem is that caused by the availability

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in some areas of reduced rates when drafts of more than a certain number of men are sent at one time. A solution of the problem using the assumption that advantage is taken of all such available group rates can be justified by assuming that enlistees can be told to report for transportation a long enough time after first enlisting to ensure that the station has enough recruits to obtain group rates. It should be noted that various standard queueing theory models of this situation might be used to select appropriate delays.

The input used in the problems solved for this paper were obtained as follows:

Costs of air, train, and bus tickets from each RS to each RTC (San Diego and Great Lakes) were requested from each RS. In addition, the RS's were asked to supply meal costs, miscellaneous costs, and time enroute for each mode. Not all requests were answered, so missing values were supplied by estimates. In the data supplied, the numbers of men required at the destinations and available at the origins were stated in percents, the information having been supplied by the Bureau of Naval Personnel. The letter prompting this study specified that each Recruit Training Center should receive the same number of recruits, giving the effective determination of that input as fifty percent.

Once having solved the basic problem for the various conditions of restriction on cost (maximum cost, minimum single fare, minimum group fare, minimum with time limit, etc.), it would be of interest to investigate the effect on the optimal solution when changes are made in the system. Since in the transportation problem, the addition of new variables (RS's or RTC's) requires not a variation of the same problem, but the solution of an entire new problem, such additions will not be

investigated. The changes in the system which appear to lead to fruitful investigations are changes in the costs, in the proportion of recruits assigned to each RTC, and in the numbers available at each RS. For a simplified mathematical statement of the foregoing formulation, see Appendix A.

3. Sensitivity Analysis Using Cost Inputs as Parameters.

3.1 The following assumptions are used for this analysis:

First - The statistics presented in the tables are NOT OFFICIAL NAVY statistics and the conclusions from this analysis are strictly the personal opinions of the authors.

Second - It was stated in the introduction of this paper that there are thirty-nine main RS's; however, this analysis was conducted using forty since we believed at the time that Baltimore was a main Recruiting Station. This analysis was completed prior to the discovery of this oversight. This oversight has no effect on the conclusions or the validity of the described method for solving the problem. Hence, for this analysis, recruits are sent from forty RS to two RTC's in accordance with the travel rates or cost coefficients listed in Table I, page 25. These rates are constantly changing; therefore, this table must be "up-dated" to be applicable to a particular time. Computed in these rates are subsistence which includes meals, berthing, limousine service, etc., and pay during travel which amounts to twelve cents per hour.

Third - The overall annual Navy recruiting quota is distributed among the forty Recruiting Stations as listed in Table II, page 26. These percentages were computed by multiplying the annual district quota by the individual Recruiting Station's annual quota. For example: The first recruiting district is responsible for 16.52% of the overall annual Navy quota. Of this percent, Boston is responsible for 27.00% within the first district; therefore, Boston is responsible for 27.00 times 16.52 or 4.46% of the overall annual Navy quota. These percentage assignments are redistributed slightly each year; therefore, this table also should be "up-dated" to be applicable to a particular year.



Furthermore, some of the RS's have class "A" substations which further breaks the percentage down; however, for the problem discussed in this paper only the forty main recruiting stations were considered as origins because the percentage breakdowns to the class "A" substations was not available. Again, this does not change the basic method of solving the problem.

Fourth - It is emphasized most emphatically that this analysis is based only on costs incurred during transportation and ignores any consideration of costs incurred before arrival of candidates at RS's (i.e., Recruiting Stations operating costs) or after arrival at RTC's (i.e., costs of operating the RTC's).

The sensitivity analysis was conducted by using a fixed set of cost input parameters and varying the percent recruit input to RTC Great Lakes from 0% to 100%. Since the total of 100% recruits is sent to RTC Great Lakes and RTC San Diego, it is evident that RTC San Diego must receive the complement of those recruits sent to RTC Great Lakes. When this approach was used a particular sequence of recruiting stations and corresponding percentages resulted; these percentages were the levels of input to RTC Great Lakes at which the corresponding RS's commenced sending recruits to RTC Great Lakes. On each of the following four pages the parameter of cost input is defined. Below each is tabulated the applicable sequence of RS's.

Max Cost Inputs: Max Cost Inputs are defined as the maximum transportation costs, regardless of mode, between a RS and the two Recruit Training Centers. These inputs were used to put an upper bound on the costs. The use of these cost inputs resulted in transporting the recruits via all three modes; however, each recruit traveled via only a single mode.

Sequence of RS's and the Corresponding Percentage at
Which Each RS Commences to Send Recruits to RTC Great Lakes

New York City	1%	Detroit	40%	Raleigh	68%
Milwaukee	8%	Louisville, Ky	44%	Des Moines	70%
Chicago	10%	Columbus	46%	New Orleans	72%
Cleveland	14%	Jacksonville, Fla	47%	Kansas City	73%
Indianapolis	17%	Richmond, Va	50%	Oklahoma City	76%
Baltimore	19%	Boston	51%	Houston	77%
Albany	20%	Nashville	55%	Dallas	79%
Pittsburgh	23%	Cincinnati	58%	Denver	81%
Philadelphia	26%	St. Louis	59%	Albuquerque	83%
Minneapolis	31%	Omaha	62%	Seattle	85%
Ashland, Ky	34%	Little Rock	64%	Portland	87%
Buffalo	36%	Macon, Ga	65%	San Francisco	90%
Washington, DC	38%	Birmingham	66%	Los Angeles	94% - 100%
Columbia, S.C.	39%				

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Air Cost Inputs: Air Cost Inputs are defined as the cost to transport all recruits only by air. These inputs were used primarily to arrive at a cost figure when minimum time enroute is the primary consideration. For Chicago, where no air transportation is available to RTC Great Lakes, the train fare was used under the assumption that time spend enroute was less than by bus.

Sequence of RS's and the Corresponding Percentage at
Which Each RS Commences to Send Recruits to RTC Great Lakes

Albany	1%	Indianapolis	44%	New Orleans	69%
Louisville	3%	Baltimore	45%	Omaha	70%
Cleveland	4%	Columbus	47%	Kansas City	72%
New York City	7%	Nashville	48%	Little Rock	74%
Boston	15%	Cincinnati	50%	Oklahoma City	76%
Ashland	19%	Raleigh	52%	Dallas	77%
Richmond	21%	Columbia	53%	Houston	79%
Philadelphia	22%	Jacksonville	55%	Denver	81%
Detroit	27%	St. Louis	57%	Albuquerque	83%
Milwaukee	31%	Macon	60%	Seattle	85%
Washington, DC	33%	Birmingham	61%	Portland	87%
Chicago	34%	Des Moines	63%	San Francisco	90%
Pittsburgh	38%	Minneapolis	65%	Los Angeles	94% - 100%
Buffalo	42%				

MinT Cost Inputs: MinT Cost Inputs are defined as the minimum costs allowing for a maximum time enroute of thirty-six hours. Thirty-six hours is an arbitrary value assumed to be realistic by the authors and any other value might be used to obtain comparable results. With these cost inputs, as with the Max cost inputs, the transporting of recruits via all three modes resulted.

Sequence of RS's and the Corresponding Percentage at
Which Each RS Commences to Send Recruits to RTC Great Lakes

Ashland, Ky	1%	Washington, DC	41%	Little Rock	69%
Boston, Mass	2%	Macon	42%	New Orleans	70%
Louisville, Ky	6%	Indianapolis	43%	Omaha	71%
Richmond	8%	Buffalo	45%	Kansas City	73%
Philadelphia	9%	Chicago	47%	Denver	76%
New York City	14%	Birmingham	52%	Oklahoma City	77%
Albany	21%	Cincinnati	54%	Dallas	79%
Cleveland	24%	Jacksonville	55%	Houston	81%
Detroit	27%	St. Louis	58%	Albuquerque	83%
Baltimore	31%	Raleigh	61%	Portland	85%
Milwaukee	33%	Columbia	62%	San Francisco	87%
Columbus	35%	Des Moines	63%	Seattle	92%
Nashville	36%	Minneapolis	65%	Los Angeles	94% - 100%
Pittsburgh	38%				

The following table shows the results of the experiments conducted on the 10th of May 1900. The experiments were conducted on the 10th of May 1900. The results of the experiments are as follows:

Experiment 1					
10	10	10	10	10	10
20	20	20	20	20	20
30	30	30	30	30	30
40	40	40	40	40	40
50	50	50	50	50	50
60	60	60	60	60	60
70	70	70	70	70	70
80	80	80	80	80	80
90	90	90	90	90	90
100	100	100	100	100	100
110	110	110	110	110	110
120	120	120	120	120	120
130	130	130	130	130	130
140	140	140	140	140	140
150	150	150	150	150	150
160	160	160	160	160	160
170	170	170	170	170	170
180	180	180	180	180	180
190	190	190	190	190	190
200	200	200	200	200	200

Bus Cost Inputs: Bus Cost Inputs are defined as the costs to transport all recruits by bus. No maximum time enroute was considered with these inputs. We realize that it is not feasible to order a recruit to ride a bus for seventy-eight hours from Portland, Maine to RTC San Diego, however, these inputs also correspond to the minimum costs available and thus, put a lower bound on the costs.

Sequence of RS's and the Corresponding Percentage at
Which Each RS Commences to Send Recruits to RTC Great Lakes

New York City	1%	Columbus	44%	Birmingham	68%
Philadelphia	8%	Ashland	46%	Omaha	70%
Milwaukee	13%	Louisville	47%	New Orleans	72%
Chicago	15%	Macon	49%	Kansas City	73%
Pittsburgh	19%	Buffalo	50%	Oklahoma City	76%
Baltimore	22%	Columbia	52%	Dallas	77%
Indianapolis	24%	St. Louis	54%	Houston	79%
Detroit	25%	Raleigh	56%	Denver	81%
Cleveland	30%	Albany	58%	Albuquerque	83%
Minneapolis	33%	Des Moines	60%	Portland	85%
Washington, DC	36%	Nashville	62%	Seattle	87%
Richmond	37%	Jacksonville	64%	San Francisco	90%
Cincinnati	38%	Little Rock	67%	Los Angeles	94% - 100%
Boston	40%				

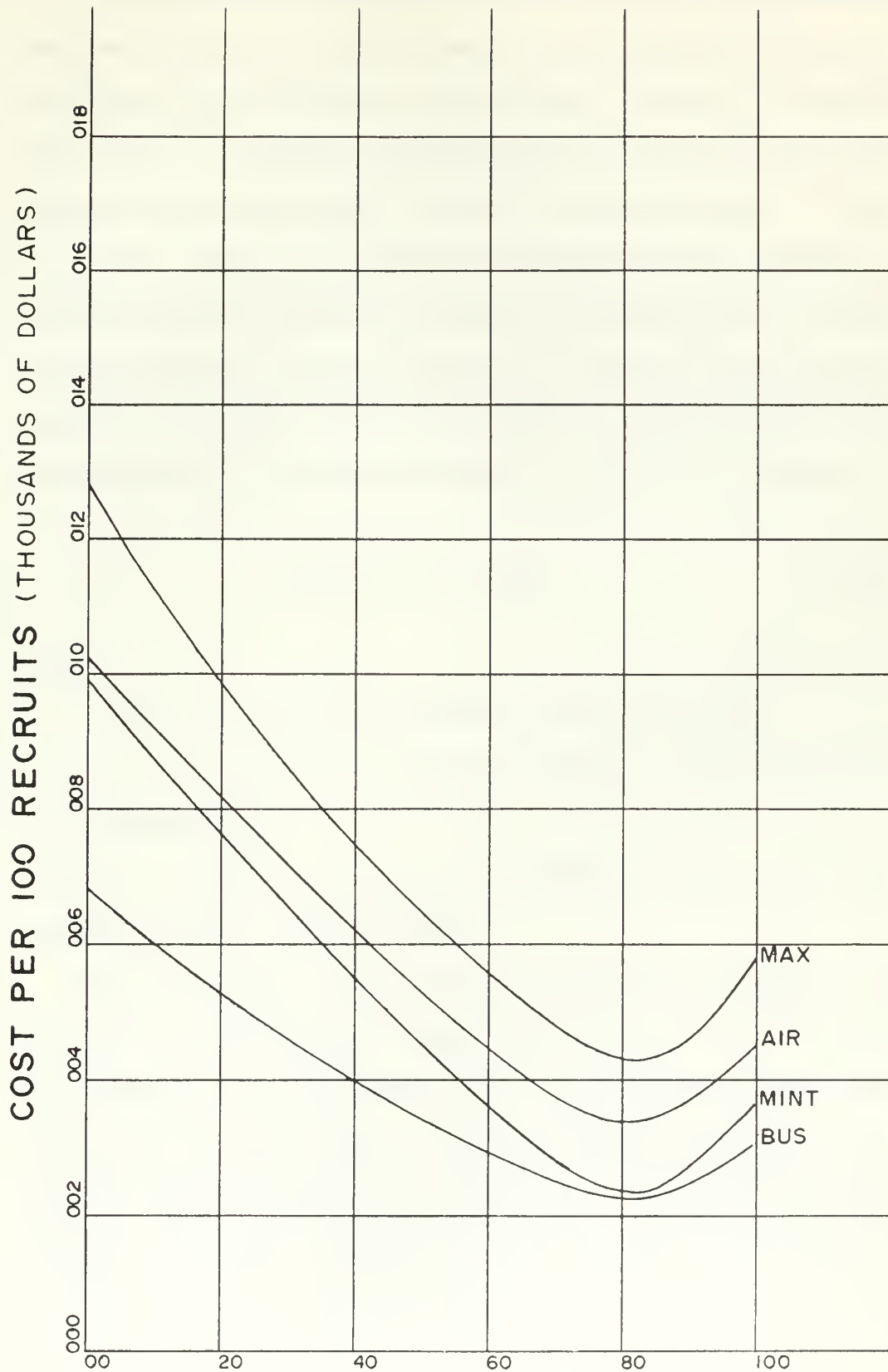
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 possible to describe it in a few words.

Year	Month	Day	Time	Location	Event
1900	Jan	1	10:00	London	Meeting of the Council
1900	Jan	2	10:00	London	Meeting of the Council
1900	Jan	3	10:00	London	Meeting of the Council
1900	Jan	4	10:00	London	Meeting of the Council
1900	Jan	5	10:00	London	Meeting of the Council
1900	Jan	6	10:00	London	Meeting of the Council
1900	Jan	7	10:00	London	Meeting of the Council
1900	Jan	8	10:00	London	Meeting of the Council
1900	Jan	9	10:00	London	Meeting of the Council
1900	Jan	10	10:00	London	Meeting of the Council
1900	Jan	11	10:00	London	Meeting of the Council
1900	Jan	12	10:00	London	Meeting of the Council
1900	Jan	13	10:00	London	Meeting of the Council
1900	Jan	14	10:00	London	Meeting of the Council
1900	Jan	15	10:00	London	Meeting of the Council
1900	Jan	16	10:00	London	Meeting of the Council
1900	Jan	17	10:00	London	Meeting of the Council
1900	Jan	18	10:00	London	Meeting of the Council
1900	Jan	19	10:00	London	Meeting of the Council
1900	Jan	20	10:00	London	Meeting of the Council
1900	Jan	21	10:00	London	Meeting of the Council
1900	Jan	22	10:00	London	Meeting of the Council
1900	Jan	23	10:00	London	Meeting of the Council
1900	Jan	24	10:00	London	Meeting of the Council
1900	Jan	25	10:00	London	Meeting of the Council
1900	Jan	26	10:00	London	Meeting of the Council
1900	Jan	27	10:00	London	Meeting of the Council
1900	Jan	28	10:00	London	Meeting of the Council
1900	Jan	29	10:00	London	Meeting of the Council
1900	Jan	30	10:00	London	Meeting of the Council
1900	Jan	31	10:00	London	Meeting of the Council

3.2 A graph of percent recruit input to RTC Great Lakes vs. transportation costs per one-hundred recruits is given in Figure I, page 14. This graph illustrates how the variation of recruit input between the two RTC's affects transportation costs. The curves on the graph were plotted by using the computed data found in Tables III and IV, pages 27 and 28 respectively.

3.3 A FORTRAN listing of the linear program used for the previous sensitivity analysis is given in Appendix E. This listing is included as an aid for those who desire to pursue this transportation problem further; a detailed explanation of this program is not considered essential to the results of this paper. A format of the computer print-out of the above program for a 5% interval, starting at 30% recruit input to RTC Great Lakes, and using 1% increments and Air cost inputs is given in Tables VI-A through VI-F, pages 30 to 35. Table V, page 29, presents a number code which correlates the P-vectors in Tables VI-A through VI-F, pages 30 to 35, to their respective RS's and RTC's.



PERCENT RECRUIT INPUT TO GREAT LAKES



4. Statistical Variation.

In the previous formulation of our problem, the basic assumption was that the inputs to the problem were known perfectly, but it is quite likely that this will not be the case. Indeed, it is apparent that there is a positive probability that the number of men enlisted at any RS will not be equal to that RS's assigned quota. In other words, the number of men to be transported from any RS may be regarded as a random variable. Regarding this random variable, β , as a parameter to the problem, we note that Garvin [3] points out that for any parameter, B , whose probability density (or mass) function is known, the minimum of the objective function, z_0 , obeys the relation

$$E[z_0(B)] \leq z_0(E[B]) \quad (4.1)$$

where:

$E[B]$ is the vector of expected values of the β 's.

$z_0(B) = \min z(b)$, the minimum objective function expressed as a function of B .

The lack of equality above makes the problem of saying something meaningful about z more difficult.

Even if we could easily find the value in the left side of equation (4.1), we would have an expression only for an ideal situation which cannot exist in the real world. In fact what we wish to know is what policy we should choose in order to approach this ideal more closely than by the use of any other policy. One approach to such problems is to find an estimator for the parameter B , and then to use this estimator, \hat{B} , to define our policy.

In attempting to find the optimum estimator, \hat{B} , we have made certain assumptions concerning the uncertain (i.e., statistically varying) value of B , which is a vector representing the number of enlistments at each RS plus the number of enlistees going to one of the RTC's. These assumptions and the models arising therefrom are discussed fully in Appendix B.

It was found (see Appendix B) that there is no simple analytical solution to the problem of determining a best estimator. The problem of even evaluating any given estimator by analytical means is so great as to be impractical to solve; so the authors have taken the approach of using the same assumptions in devising a Monte Carlo simulation. As discussed more fully in Appendix B, it is assumed that the number of people arriving at a recruiting station follow the Poisson probability law and that the number of those who actually enlist is governed by the binomial probability law. In our simulation, this random amount is determined for each RS and the problem solved to find the least cost for that set of inputs. The program is designed to solve any desired number of such subproblems in order to find an approximation to the value of the left side of equation (4.1).

The next assumption we make is that there is, conceptually, some loss in not solving the sub-problems this way, and that by following a fixed policy we will have to incur this loss at least part of the time. By the use of an estimator we can find a policy. The policy is the transportation pattern developed by solution to a sub-problem in which the estimator that we choose is used as the input. When this policy is applied to the sample of sub-problems we obtain, for each sub-problem, a difference in cost which will be either zero or more

and which is the loss for that particular sub-problem. In our Monte Carlo simulation we have evaluated only one estimator, the average B. Of course, given any other estimator, we could evaluate it also. Having found the losses we can find the average loss, which is termed sample risk. We can also find the sample deviation of loss, thereby gaining some idea of how much any particular loss may vary from this average. With larger and larger numbers of sub-problems we can more closely approach the "true" values of these statistics. The results of the Monte Carlo simulation are discussed in section 5.3.

Another possible estimator which might be used under certain circumstances is discussed in Appendix D.

5. Results and Discussion.

5.1 It is interesting to observe that the sequence of the RS's changed when the cost inputs were changed for some reason, such as changing the mode of travel (air to bus), or changing the criterion for choosing a particular mode of travel (limited time enroute).

Using Max Cost Inputs, it is noted that when the input to RTC Great Lakes is changed to 1%, New York is the first RS to send recruits to RTC Great Lakes, and it sends 1% of its quota. New York's total quota is 7.35%; therefore, as the input to RTC Great Lakes is increased by 1% increments, New York increases its quota by 1% increments until its total quota is sent there. One explanation for this could be that New York is responsible for the largest overall Navy quota. However, referring to Table I, page 25, it is observed that the DIFFERENCE in Max costs for New York is $\$211.02 - \$44.96 = \$166.06$. This difference is larger than for any other city; therefore, it would seem that this combination would give the greater savings.

As the recruit input to RTC Great Lakes continues to change by 1% increments it is noted that when the input reaches 8%, Milwaukee is the next RS to change its distribution and commence sending recruits to RTC Great Lakes. Why should Milwaukee send recruits to RTC Great Lakes before Chicago? Once again it is observed that the DIFFERENCE in Max costs for Milwaukee is $\$152.99 - \$10.09 = \$142.90$, compared to $\$143.39 - \$1.37 = \$142.02$, for Chicago. It is interesting to note that the input to RTC Great Lakes must reach 10% before Chicago starts sending recruits to Great Lakes. Although Chicago is the closest recruiting station to Great Lakes, this does not imply that the minimum distance traveled is going to give maximum savings.

Again referring to Table I, page 25, only this time using the Air or Bus costs, it is observed that the first RS to commence sending recruits to both RTC's is not New York, but that city that has the greatest DIFFERENCE in the appropriate costs. When that RS has commenced sending its total quota to RTC Great Lakes, the next city with the greatest DIFFERENCE in costs commences to send recruits to both RTC's. This method continues until all RS's are sending their total quota to RTC Great Lakes. Therefore, when the problem is linear and involves two destinations, as in this problem, we see that it is the DIFFERENCE in costs of the applicable mode of travel to the two destinations that is the controlling factor.

5.2 The graph in Figure 1, page 14, illustrates that there is a decrease in travel costs up to an 82% input of recruits to RTC Great Lakes and then the costs begin to increase. In the case of Max and MinT cost inputs, where it is recalled that mixed travel modes were used, the minimum occurred at roughly the same percentage. Thus, the decrease occurs regardless of the mode of transportation.

Why should the minimum costs occur at such a high percentage input to RTC Great Lakes? It can be conjectured that one possible explanation might be:

1. It costs less to transport recruits from all parts of the country to RTC Great Lakes than to RTC San Diego because RTC Great Lakes is geographically more centrally located.
2. The more populous areas are responsible for a larger overall annual recruiting quota and the majority of these areas are located nearer RTC Great Lakes than RTC San Diego.

We would like to emphasize at this point that we have discussed only that the transportation costs to RTC Great Lakes are less than those to RTC San Diego up to a certain percentage of recruit input. We have made no cost analyses of sending recruits through the two Recruit Training Centers.

5.3 In testing the Monte Carlo approach to this problem, first a sample of fifty randomly generated B-vectors were analyzed. The inputs to the random vector generator sub-program were,

- 1) RS quotas expressed as integer numbers,
- 2) the average number of interviews held per day at each RS, and
- 3) the individual probability of success (obtaining an enlistment) per interview for each RS.

Values of the last two parameters were quite arbitrarily chosen, the average interview number being made approximately a fourth more than the annual quota divided by 365. The probability of success at an interview was chosen arbitrarily over a range from 55% to 80%. A table of these inputs is given in Table VII-B. The vectors resulting had entries of a fair number of stations (about half in most cases) which fulfilled their quota. A typical vector resulting is shown in Table VII-C. The other percent-of-quotas-enlisted ranged down to 64%, but the total quota was filled to a level never less than 93% and never as great as 95% in the fifty samples. Further, the solution pattern of transportation was the same for each random vector, for these arbitrary parameter values. Typical results of the solution of one vector are given in Table VII-D. In short, the solution was quite insensitive to changes generated in a small sample by these parameter values, and the sample deviation of the random variable β for the RTC was small. Because

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of this result, it was decided that more runs would be desirable to determine whether different parameter values might lead to very different results. Accordingly, four samples were computed, with fifteen random vectors each and with combinations of parameters that differed greatly. The average number of interview for each RS was given the value of the quota of that RS divided by 365 for two runs and twice that much for two others. The probability of interview success was fixed by 55% for all stations for two runs and 65% for the other two runs. These were, of course, combined in such a way as to get all four combinations.

Finally, our simulation also produced sample mean loss, an approximation to the risk for the given estimator, and the sample deviation of that loss. Examples of these results are included in Table VII-D. When low averages and low probabilities (55%) were used, the percent of total quota filled never dropped below 56% and never reached 57%. Individual percent of quotas ranged from 39% to 73%. When low averages and high probabilities (75%) were used the percent of total quota filled ranged between 75% and 77% while individual quotas were filled to levels from 55% to 96%. For the combination of high average number of arrivals and low probability of enlistments total quota fulfillment never fell below 99% while individual station quotas were filled to levels as low as 80% for the very few in each sample not fulfilling their quotas. The combination of high probability and high average gave 100% fulfillment for all fifteen samples.

The lack of variation in the percent of total quota filled can be viewed as a substantiation of the validity of the statement that for large samples β_{m+1} can be approximated by a normal random variable independent of the other m random variables.

This lack of variation had one other notable effect. The mean B vector, \bar{B} , proved to be a very good estimator; there were, in fact, no samples on which the result was non-optimal. It should be noted that should the use of the transformation \bar{P}^{-1} result in a sample with a non-optimal solution, our program will compute the loss on that sample. Therefore, not only was \bar{B} a good estimator for our selected inputs, but it is also obvious that for these inputs, a static model is adequate. Of course, use of different quotas and different parameters might cause the probability mass functions governing the inputs to "spread" so much that non-optimal solutions might result in enough cases to make a dynamic model desirable.

5.4 In the probability model developed herein many simplifying assumptions were made which might have an effect on the conclusions. Some of them are discussed below.

1. It was assumed that quotas assigned to RS's were fixed. It has been learned that such quotas may in fact be changed during the year in order to capitalize on peculiar seasonal or regional situations effecting recruitment.
2. It was assumed that the number of interviews held by a recruiting station could be described by a Poisson probability mass function. In using the Poisson distribution, one assumes that the probability of more than one event occurring in time Δt is of smaller order than Δt so that as Δt approaches zero the probability of two or more events occurring in Δt also is negligible. In practice recruiters often give talks to large groups of high school or college students. This might

be said to be more than one interview; however, it is believed that the prospective enlistees must still enter a queue of some sort before making a commitment to enlist, so that the mass lecture may be regarded as one of the factors determining α , the average number of interviews for the j^{th} RS rather than as an actual multiple interview.

3. Probabilistic independence of the occurrence of interviews was assumed. Arguments contrary to this assumption have been acknowledged above and some defense given. As a further defense, we note that the amount of personal effect required of the recruiter is a factor tending to promote the statistical independence of these random variables.
4. It has been assumed that the parameters of the Poisson and binomial distributions used could be determined for each RS. They cannot be completely determined, of course, but it is likely that enough data can be obtained from RS records to arrive at very good estimates.

Although some of these objections are forceful, none is believed to be strong enough to invalidate the usefulness of the results.

6. Conclusions.

It can be concluded from this investigation that:

1. We have been able to devise a method enabling us to simulate (at least approximately) the uncertainties caused by the probabilistic nature of numbers of enlistments and thus, can consider our model to be more than a mere laboratory affair.
2. We have been able to develop a rationale for the derivation of an estimator and have been able to obtain an approximation of the success we may expect in solving our problem relative to an ideal solution, given any particular estimator.
3. By means of simulation, we have found that the mean input vector, \bar{B} , is an estimator which yields optimal solutions for a very large number of samples and for widely varying parameters. We have seen that this indicates the adequacy of a static model.
4. We have indicated another promising estimator that might be developed that could have considerable appeal to a planner under certain circumstances.
5. We have been able to devise a method allowing us to present to the planner an assessment of the cost of certain decisions, namely the choice of assigning various quotas to each of the two RTC's, the choice of putting a limit on travel time, and the choice of an estimator.
6. We have found the simple principle underlying the choice of routes to be used by the RS's, that of maximum cost DIFFERENCE.

TABLE I

COST COEFFICIENTS FROM RECRUITING STATIONS TO RECRUIT TRAINING CENTERS

GLAKES SDIEGO	GLAKLS SDIEGO	GLAKES SDIEGO	GLAKES SDIEGO	GLAKES SDIEGO
ALBANY 52.23 161.92 74.18 203.30 52.45 98.12	ALBUQUERQUE 69.82 40.09 76.59 55.77 42.72 31.31	ASHLAND 37.14 138.40 27.56 149.27 16.70 70.82	BALTIMORE 39.33 133.42 79.97 214.65 28.08 98.01	BIRMINGHAM 42.11 118.44 37.73 108.94 22.60 60.37
BOSTON 52.34 154.46 86.83 176.98 34.27 90.20	BUFFALO 24.98 119.55 36.27 145.30 23.13 74.90	CHICAGO 0.00 97.17 1.37 143.39 1.28 73.77	CINCINNATI 20.48 106.56 30.45 119.93 11.55 68.40	CLEVELAND 22.37 126.16 21.96 164.16 17.25 82.80
COLUMBIA 43.99 128.56 59.34 164.84 34.13 84.55	COLUMBUS 24.64 117.29 31.94 128.69 12.73 68.40	DALLAS 54.31 76.11 61.61 64.87 53.25 47.18	DENVER 55.76 57.96 51.28 68.71 27.53 30.62	DES MOINES 24.26 99.05 30.31 104.84 15.04 60.26
DETROIT 22.49 122.92 17.13 94.88 14.65 82.46	HOUSTON 66.41 76.91 71.18 102.16 41.08 53.27	INDIANAPOLIS 14.78 109.03 15.80 122.55 11.70 79.55	JACKSONVILLE 49.26 130.35 59.53 156.11 26.63 81.82	KANSAS CITY 24.22 79.71 26.78 96.35 18.72 54.56
LITTLE ROCK 43.94 91.50 33.27 124.00 20.37 59.78	LOS ANGELES 97.81 10.09 138.74 4.52 71.55 3.56	LOUISVILLE 20.78 129.95 30.78 118.89 11.79 65.55	MACON 50.77 128.44 87.66 165.41 30.97 83.10	MILWAUKEE 10.09 107.10 3.35 152.99 2.23 77.44
MINNEAPOLIS 25.24 95.70 25.92 141.94 13.92 76.01	NASHVILLE 26.53 118.18 28.55 109.04 15.50 60.01	NEW ORLEANS 44.00 103.02 59.59 131.94 35.14 72.10	NEW YORK 44.96 147.51 42.96 211.02 37.23 124.88	OKLAHOMA CITY 43.43 77.22 41.10 66.38 29.52 57.35
OMAHA 23.74 86.19 33.64 113.93 19.00 56.63	PHILADELPHIA 41.22 142.01 76.18 204.05 31.01 110.60	PITTSBURGH 29.63 124.72 51.19 178.49 22.54 94.05	PORTLAND 107.28 61.63 78.41 38.53 68.27 31.50	RALEIGH 47.09 131.87 62.00 137.01 29.76 76.86
RICHMOND 41.62 142.34 63.69 156.70 29.93 89.80	SAN FRANCISCO 91.55 20.98 128.32 27.80 63.54 14.88	SLATTLE 106.29 62.97 103.95 72.75 60.65 23.36	ST LOUIS 19.49 98.49 13.44 100.15 8.81 56.70	WASHINGTON 34.31 130.74 69.79 177.98 31.31 93.25



TABLE II

DISTRIBUTION OF OVERALL ANNUAL RECRUITING QUOTA

ALBANY	2.56	DES MOINES	1.86	NEW ORLEANS	1.24
ALBUQUERQUE	1.43	DETROIT	4.33	NEW YORK	7.35
ASHLAND	1.36	HOUSTON	2.53	OKLAHOMA CITY	1.41
BALTIMORE	1.39	INDIANAPOLIS	1.66	OMAHA	1.67
BIRMINGHAM	2.05	JACKSONVILLE	2.44	PHILADELPHIA	4.88
BOSTON	4.46	KANSAS CITY	2.50	PITTSBURGH	3.06
BUFFALO	2.14	LITTLE ROCK	1.36	PORTLAND	2.14
CHICAGO	4.56	LOS ANGELES	6.20	RALEIGH	1.47
CINCINNATI	1.67	LOUISVILLE	1.39	RICHMOND	1.39
CLEVELAND	2.92	MACON	1.35	SAN FRANCISCO	4.80
COLUMBIA	1.47	MILWAUKEE	1.92	SEATTLE	2.75
COLUMBUS	1.39	MINNEAPOLIS	3.38	ST LOUIS	2.77
DALLAS	1.83	NASHVILLE	2.05	WASHINGTON	0.83
DENVER	1.80				



TABLE III

TABULATION OF MAXIMUM TRAVEL COSTS PER RECRUIT AND
CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	128.69	26	91.04	52	63.18	78	43.76
01	127.03	27	89.76	53	62.28	79	43.45
02	125.37	28	88.49	54	61.38	80	43.31
03	123.71	29	87.22	55	60.48	81	43.16
04	122.05	30	85.95	56	59.58	82	43.03
05	120.39	31	84.75	57	58.69	83	43.01
06	118.73	32	83.59	58	57.79	84	43.22
07	117.07	33	82.43	59	56.92	85	43.54
08	115.56	34	81.28	60	56.11	86	43.88
09	114.13	35	80.16	61	55.31	87	44.23
10	112.70	36	79.07	62	54.50	88	44.69
11	111.28	37	77.99	63	53.70	89	45.15
12	109.86	38	76.90	64	52.99	90	46.15
13	108.44	39	75.85	65	52.11	91	47.16
14	107.02	40	74.81	66	51.33	92	48.16
15	105.61	41	73.81	67	50.57	93	49.17
16	104.19	42	72.81	68	49.81	94	50.23
17	102.78	43	71.80	69	49.06	95	51.51
18	101.41	44	70.80	70	48.31	96	52.80
19	100.06	45	69.81	71	47.57	97	54.09
20	98.72	46	68.83	72	46.84	98	55.37
21	97.43	47	67.87	73	46.13	99	56.66
22	96.14	48	66.90	74	45.43	100	57.95
23	94.86	49	65.93	75	44.74		
24	93.58	50	65.00	76	44.39		
25	92.31	51	64.08	77	44.07		

TABULATION OF AIR TRAVEL COSTS PER RECRUIT AND
CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	102.83	26	76.00	52	51.31	78	34.18
01	101.74	27	75.00	53	50.46	79	34.04
02	100.64	28	74.00	54	49.62	80	33.94
03	99.54	29	72.99	55	48.79	81	33.84
04	98.45	30	71.99	56	47.93	82	33.82
05	97.42	31	70.99	57	47.18	83	33.90
06	96.38	32	70.02	58	46.39	84	34.20
07	95.34	33	69.05	59	45.60	85	34.62
08	94.32	34	68.09	60	44.81	86	35.05
09	93.29	35	67.14	61	44.03	87	35.49
10	92.27	36	66.18	62	43.27	88	35.95
11	91.24	37	65.22	63	42.51	89	36.40
12	90.21	38	64.26	64	41.76	90	37.11
13	89.19	39	63.31	65	41.02	91	37.81
14	88.16	40	62.36	66	40.31	92	38.51
15	87.14	41	61.41	67	39.61	93	39.22
16	86.12	42	60.46	68	38.90	94	39.96
17	85.10	43	59.52	69	38.29	95	40.83
18	84.08	44	58.57	70	37.71	96	41.71
19	83.06	45	57.63	71	37.13	97	42.59
20	82.05	46	56.69	72	36.57	98	43.46
21	81.04	47	55.76	73	36.02	99	44.34
22	80.03	48	54.84	74	35.49	100	45.22
23	79.02	49	53.92	75	35.01		
24	78.02	50	53.02	76	34.67		
25	77.01	51	52.16	77	34.40		



TABLE IV

TABULATION OF MINT TRAVEL COSTS PER RECRUIT AND
CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	99.43	26	70.05	52	43.69	78	24.10
01	98.22	27	68.96	53	42.73	79	23.92
02	97.01	28	67.88	54	41.78	80	23.78
03	95.81	29	66.80	55	40.84	81	23.67
04	94.61	30	65.72	56	39.92	82	23.57
05	93.40	31	64.64	57	39.00	83	23.54
06	92.20	32	63.59	58	38.10	84	23.65
07	91.02	33	62.54	59	37.20	85	24.08
08	89.89	34	61.49	60	36.31	86	24.55
09	88.77	35	60.44	61	35.46	87	25.24
10	87.66	36	59.41	62	34.61	88	26.00
11	86.55	37	58.38	63	33.76	89	26.77
12	85.44	38	57.36	64	32.92	90	27.53
13	84.34	39	56.34	65	32.09	91	28.30
14	83.23	40	55.32	66	31.27	92	29.13
15	82.13	41	54.31	67	30.45	93	29.95
16	81.03	42	53.33	68	29.63	94	30.31
17	79.92	43	52.35	69	28.90	95	31.75
18	78.82	44	51.38	70	28.20	96	32.70
19	77.72	45	50.41	71	27.52	97	33.64
20	76.61	46	49.45	72	26.85	98	34.58
21	75.51	47	48.48	73	26.21	99	35.53
22	74.42	48	47.52	74	25.60	100	36.47
23	73.32	49	46.57	75	24.99		
24	72.23	50	45.61	76	24.67		
25	71.14	51	44.65	77	24.37		

TABULATION OF BUS TRAVEL COSTS PER RECRUIT AND
CORRESPONDING PERCENTAGE INPUT TO GREAT LAKES

00	68.75	26	48.60	52	33.29	78	22.87
01	67.88	27	47.92	53	32.79	79	22.74
02	67.00	28	47.24	54	32.31	80	22.62
03	66.12	29	46.56	55	31.83	81	22.51
04	65.25	30	45.90	56	31.36	82	22.48
05	64.37	31	45.25	57	30.89	83	22.50
06	63.49	32	44.59	58	30.43	84	22.61
07	62.62	33	43.97	59	29.97	85	22.95
08	61.79	34	43.35	60	29.52	86	23.32
09	61.00	35	42.73	61	29.06	87	23.69
10	60.21	36	42.11	62	28.61	88	24.07
11	59.41	37	41.50	63	28.17	89	24.44
12	58.62	38	40.91	64	27.73	90	24.92
13	57.86	39	40.34	65	27.29	91	25.41
14	57.11	40	39.78	66	26.86	92	25.90
15	56.38	41	39.22	67	26.46	93	26.38
16	55.65	42	38.66	68	26.07	94	26.91
17	54.93	43	38.10	69	25.70	95	27.59
18	54.20	44	37.54	70	25.32	96	28.27
19	53.48	45	36.99	71	24.94	97	28.95
20	52.76	46	36.44	72	24.57	98	29.62
21	52.05	47	35.90	73	24.21	99	30.30
22	51.34	48	35.36	74	23.85	100	30.98
23	50.64	49	34.84	75	23.49		
24	49.96	50	34.32	76	23.21		
25	49.28	51	33.80	77	23.01		

TABLE V

NUMBER CODE OF RECRUITING STATIONS AND RECRUIT TRAINING CENTERS

RECRUITING STATION	GLAKES	SDIEGO
ALBANY	P1	P2
BOSTON	P3	P4
BUFFALO	P5	P6
NEW YORK	P7	P8
ASHLAND	P9	P10
BALTIMORE	P11	P12
LOUISVILLE	P13	P14
RICHMOND	P15	P16
WASHINGTON	P17	P18
BIRMINGHAM	P19	P20
COLUMBIA	P21	P22
JACKSONVILLE	P23	P24
MACON	P25	P26
NASHVILLE	P27	P28
RALEIGH	P29	P30
CINCINNATI	P31	P32
CLEVELAND	P33	P34
COLUMBUS	P35	P36
PHILADELPHIA	P37	P38
PITTSBURGH	P39	P40
CHICAGO	P41	P42
DETROIT	P43	P44
INDIANAPOLIS	P45	P46
MILWAUKEE	P47	P48
ST LOUIS	P49	P50
DENVER	P51	P52
DES MOINES	P53	P54
KANSAS CITY	P55	P56
MINNEAPOLIS	P57	P58
OMAHA	P59	P60
ALBUQUERQUE	P61	P62
DALLAS	P63	P64
HOUSTON	P65	P66
LITTLE ROCK	P67	P68
NEW ORLEANS	P69	P70
OKLAHOMA CITY	P71	P72
LOS ANGELES	P73	P74
PORTLAND	P75	P76
SAN FRANCISCO	P77	P78
SEATTLE	P79	P80

TABLE VI-A

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .719862E+04

GREAT LAKES PERCENT .300E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(6)	.214760E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(43)	.365571E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(18)	.830700E+00
P(37)	.488250E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(26)	.135875E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(20)	.205443E+01
P(36)	.139500E+01
P(12)	.139941E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(44)	.682690E+00
P(46)	.166334E+01
P(48)	.192276E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

TABLE VI-B

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .709928E+04

GREAT LAKES PERCENT .310E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(6)	.214760E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(47)	.317310E+00
P(33)	.292950E+01
P(22)	.147832E+01
P(37)	.488250E+01
P(26)	.135875E+01
P(20)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(24)	.244575E+01
P(36)	.139500E+01
P(28)	.205443E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(18)	.830700E+00
P(46)	.166334E+01
P(48)	.160545E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(43)	.433840E+01

TABLE VI-C

PROBLEM. RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .700227E+04

GREAT LAKES PERCENT .320E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR COEFFICIENT (X-ZERO COMPONENT)

P(1)	.256060E+01
P(3)	.446040E+01
P(47)	.131731E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(18)	.830700E+00
P(33)	.292950E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(20)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(28)	.205443E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(6)	.214760E+01
P(46)	.166334E+01
P(48)	.605450E+00
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(43)	.433840E+01



TABLE VI-D

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .690549E+04

GREAT LAKES PERCENT .330E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR COEFFICIENT (X-ZERO COMPONENT)

P(1)	.256060E+01
P(3)	.446040E+01
P(47)	.192276E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(12)	.139941E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.394550E+00
P(20)	.205443E+01
P(43)	.433840E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(22)	.147832E+01
P(40)	.306900E+01
P(42)	.456274E+01
P(6)	.214760E+01
P(46)	.166334E+01
P(18)	.436150E+00
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.186708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01



TABLE VI-E

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .680941E+04

GREAT LAKES PERCENT .340E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR	COEFFICIENT (X-ZERO COMPONENT)
P(1)	.256060E+01
P(3)	.446040E+01
P(41)	.563850E+00
P(7)	.735140E+01
P(9)	.136107E+01
P(47)	.192276E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.830700E+00
P(20)	.205443E+01
P(22)	.147832E+01
P(24)	.244575E+01
P(37)	.488250E+01
P(28)	.205443E+01
P(43)	.433840E+01
P(32)	.167400E+01
P(26)	.135875E+01
P(36)	.139500E+01
P(30)	.147832E+01
P(40)	.306900E+01
P(42)	.399889E+01
P(12)	.139941E+01
P(46)	.166334E+01
P(6)	.214760E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01



TABLE VI-F

PROBLEM RECRUIT *SAMPLE*

MINIMUM COST OF OBJECTIVE FUNCTION IS .671361E+04

GREAT LAKES PERCENT .350E+02

BASIS VECTORS AND COEFFICIENTS

VECTOR COEFFICIENT (X-ZERO COMPONENT)

P(1)	.256060E+01
P(3)	.446040E+01
P(4)	.156385E+01
P(7)	.735140E+01
P(9)	.136107E+01
P(47)	.192276E+01
P(13)	.139941E+01
P(15)	.139941E+01
P(17)	.830700E+00
P(37)	.488250E+01
P(43)	.433840E+01
P(24)	.244575E+01
P(26)	.135875E+01
P(28)	.205443E+01
P(30)	.147832E+01
P(32)	.167400E+01
P(20)	.205443E+01
P(36)	.139500E+01
P(22)	.147832E+01
P(40)	.306900E+01
P(42)	.299889E+01
P(12)	.139941E+01
P(46)	.166334E+01
P(6)	.214760E+01
P(50)	.277732E+01
P(52)	.180803E+01
P(54)	.186418E+01
P(56)	.250429E+01
P(58)	.338023E+01
P(60)	.167327E+01
P(62)	.143260E+01
P(64)	.188708E+01
P(66)	.253916E+01
P(68)	.136344E+01
P(70)	.124488E+01
P(72)	.141284E+01
P(74)	.620100E+01
P(76)	.214650E+01
P(78)	.480180E+01
P(80)	.275070E+01
P(33)	.292950E+01

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APPENDIX A

BASIC SYMBOLOGY OF THE TRANSPORTATION PROBLEM

It is desired to ship a homogeneous product in integral amounts, a_i ($i = 1, \dots, m$) from each of m origins and to have it arrive at n destinations in such a manner that each destination receives an amount, b_j ($j = 1, \dots, n$). The cost c_{ij} of sending a unit amount from the i^{th} origin to the j^{th} destination is known for all ij combinations. Here the total to be sent equals the total demanded, i.e.,

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

The problem is to select the amounts x_{ij} to be shipped from the i^{th} origin to the j^{th} destinations in such a way that the total cost is minimized. There can be no shipments from destinations to origins, which is to say $x_{ij} \geq 0$ for all ij . Symbolically, we may state the problem

letting:

$$z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (\text{A-1})$$

where $x_{ij} \geq 0$ for all i and j

minimize z subject to the restraints

$$\begin{aligned} \sum_{i=1}^m x_{ij} &= b_j & j &= 1, \dots, n \\ \sum_{j=1}^n x_{ij} &= a_i & i &= 1, \dots, m \end{aligned} \quad (\text{A-2})$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

Restating the transportation problem as a standard linear program in the matrix algebra form

$$\text{minimize} \quad z = C^T X \quad (A-1')$$

subject to the restraint

$$\text{where} \quad AX = B \quad (A-2')$$

$$C^T = [c_{ij}],$$

a transposed vector of c_{ij} 's (all vectors here are assumed to be vertical arrays, hence, any horizontal array is a transpose).

$$X = [x_{ij}]$$

$$A = \begin{bmatrix} 1100 & . & . & . & 0 \\ 0011 & . & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 00 & . & . & . & 11 \\ 1010 & . & . & 101 \end{bmatrix} \quad (A-3)$$

the matrix of coefficients of the system of equations:

$$\sum_j x_{ij} = a_i \quad i = 1 \dots m$$

$$\sum_i x_{i1} = b_1$$

we delete the equation for b_2 , since any one of the equations may be considered redundant, and

$$B = \begin{bmatrix} a_i \\ b_j \end{bmatrix}$$

the adjoined vectors of supplies and demands.

The investigations of sensitivity can be stated in the following ways:

I. Variations in C^T

Assuming the already-found minimum basic feasible solution is X_0 , since it remains a minimum basic feasible solution if and only if

$$z_j \leq C_j \quad \text{for all } j$$

where

$$X_0 = P^{-1}P_0$$

$$X_j = P^{-1}P_j$$

and P^{-1} is the inverse of P , the $(m+n) \times (m+n)$ matrix composed of column vectors of A .

Then it is true that if

$$(\Delta \bar{C})^T X_j - \Delta C_j \leq z_j - C_j$$

the solution X_0 remains minimum, i.e., in investigating the effect of cost on the minimum we first solve (A-1) to see if our old solution is still good. If not, we must resolve.

II. Changes in B

Changes in B , principally in the partition of it involving the amounts to be supplied, are of the most interest in this problem since

there is some probability that individual recruit stations will not meet their quotas.

Since the altered B , B' may be described $B' = B + \Delta B = P'_0$, changes in the minimum basic feasible solution satisfy

$$X'_0 = X_0 + \Delta X_0$$

$$X'_0 = P^{-1} P_0$$

and X'_0 will be a basic feasible solution, if and only if

$$x'_j > 0 \quad \text{for all } x'_j \text{ members of } X'_0.$$

The changed minimum cost (objective function) z'_0 is

$$\begin{aligned} z'_0 &= \bar{C}^T X'_0 \\ &= z_0 + \bar{C}^T \Delta X_0 \end{aligned}$$

$$\text{i.e.,} \quad \Delta z_0 = \bar{C}^T \Delta X_0 \quad (\text{A-3})$$

Since the values of B are subject to statistical variation, the planner responsible for budgeting transportation will want to know how such variation will affect his cost, and equation (A-3) gives an indication of how to go about it, as we observe later in Chapter 4, Statistical Variation. For a complete discussion of sensitivity analysis, see Garvin [3].

APPENDIX B

PROBABILISTIC FORMULATION

In this discussion, extensive use will be made of the symbols and ideas presented in Appendix A and some new ones will be introduced. A full list of symbol definitions is presented in the Glossary, Appendix B.

In the problem under consideration we are aided in our probability investigations by the fact that of the system of equations describing the transportation problem, there is always one redundant equation which may be discarded. Since the inputs from the RS's are the members of B of most interest, we discard an equation which describes the input to one of the RTC's. We now assume that the distributions of the RS outputs, $F_{\beta}(b)$, are independent and of the same form although they have different parameters. Of course, the assumption of independence may be attacked, as it implies that the events being assigned the numerical values b , i.e., the number of enlistments at the RS's, do not depend on any common phenomenon such as prosperity, war fever, etc. (We do not include the effect of seasonal fluctuations since, although they probably affect the rate of enlistment over time intervals shorter than a year, they probably have no effect on the annual rate).

A complication of notation is that when we formulate the problem in the simplex notation, we note that

$$B = [\beta_j] \text{ corresponds to } \begin{bmatrix} a_1 \\ \vdots \\ a_m \\ b_j \end{bmatrix} \text{ of the}$$

transportation notation and the b_j of the latter are restricted so that

$$\sum_i a_i = \sum_j b_j$$

or in the notation of our probabilistic simplex

$$\sum_{i=1}^m \beta_i = \sum_{j=m+1}^{m+n} \beta_j$$

Fortunately in this problem (as indicated above) $n = 2$ and since the $(m + n)^{\text{th}}$ equation is redundant and is discarded, there is only one random variable β_j , namely β_{m+1} , that is a function of the others.

The vector B becomes

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_j \\ \vdots \\ \beta_m \\ \beta_{m+1} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_j \\ \vdots \\ \beta_m \\ \gamma \sum_{j=1}^m \beta_j \end{bmatrix} .$$

Since we know the proportion γ , of the total number of recruits $\sum \beta_j$ which the first Recruit Training Center is assigned, we have written

$$\beta_{m+1} = \gamma \sum_{j=1}^m \beta_j .$$

In order to obtain the form of the probability distribution function for β_j , $j = 1, \dots, m$ the following reasoning was used:

A recruiter has some probability, p , of enlisting any candidate he interviews. If these probabilities are assumed identical for any one Recruiting Station, the number enlisted in N interviews is a random variable described by a binomial distribution with parameters N and p . However, the number of interviews held is largely out of the

control of the station, that is, it is controlled in only a general way and is not predictable. Energetic RS personnel can raise the local level of interest but must wait for candidates to show that interest. If time periods are properly chosen it may be said that the number of people arriving to be interviewed at an RS is a random variable obeying the following restrictions:

1. The probability of one arrival in a period Δt in length is roughly proportional to the average rate of arrivals and to the length of Δt .
2. The probability of zero arrivals in a period Δt long is roughly one minus the probability for one arrival.
3. The probability of two or more arrivals in a period Δt is of a much smaller order than those in 1 and 2.
4. The arrival of any one person is independent of that of any other.

These are the standard assumptions for the establishing of a Poisson probability distribution law, and the steps used in arriving at it are discussed in elementary probability texts (vide Parzen [5]). We will simply say that N (the number of interviews held in a RS in some convenient time interval Δt) is described by the mass function

$$P [N = n] = e^{-\alpha t} \frac{(\alpha t)^n}{n!} \quad n = 0, 1, 2, \dots$$

$$= 0 \quad \text{otherwise}$$

where α is the average number of arrivals per period (e.g., per recruiting day).

Now the number of men actually enlisted in N interviews is seen to be a random variable described by a conditional distribution function.

For the j^{th} station

$$F_{\beta_j | N_j} (b_j | n_j) = P[\beta_j \leq b_j | N = n]$$

There is one more complicating detail, which is that the recruiter is not able to stop recruiting until he has filled his quota, q_j . In effect, this truncates the binomial distribution governing the "output" of his n_j interviews, so that

$$0 \leq \beta_j \leq n_j \quad \text{if } n_j \leq q_j$$

and

$$0 \leq \beta_j \leq q_j \quad \text{if } q_j < n_j$$

Dropping the j subscripts for clarity, then

$$\begin{aligned} F_{B|N} (q | n) &= \sum_{\{b | p(b|n) > 0\}} p(b|n) \\ &= K_1 \sum_{i=0}^n \binom{n}{i} p^i r^{n-i} = 1 \quad \text{if } n \leq q \\ &= K_2 \sum_{i=0}^q \binom{n}{i} p^i r^{n-i} = 1 \quad \text{if } q < n \end{aligned}$$

so that $K_1 = 1$

and
$$K_2 = \frac{1}{\sum_{i=0}^q \binom{n}{i} p^i r^{n-i}}$$

where $r = (1 - p)$

$$\begin{aligned}
{}^{\text{now}} F \beta^{(b)} &= \sum_{\{n | p(n) > 0\}} \beta^{(b|n)} p_N(n) \\
&= \sum_{n=0}^q (\alpha t)^n \frac{e^{-\alpha t}}{n!} \sum_{i=0}^b \binom{n}{i} p^i r^{n-i} \\
&\quad + \sum_{n=q+1}^{\infty} (\alpha t)^n \frac{e^{-\alpha t}}{n!} \frac{\sum_{i=0}^b \binom{n}{i} p^i r^{n-i}}{\sum_{i=0}^q \binom{n}{i} p^i r^{n-i}} \\
&= \sum_{i=0}^b \left[\sum_{n=0}^q (\alpha t)^n \frac{e^{-\alpha t}}{n!} \binom{n}{i} p^i r^{n-i} + \frac{\sum_{n=q+1}^{\infty} (\alpha t)^n \frac{e^{-\alpha t}}{n!} \binom{n}{i} p^i r^{n-i}}{\sum_{k=0}^q \binom{n}{k} p^k r^{n-k}} \right]
\end{aligned}$$

The size and complexity of this expression suggests the desirability of resorting to a computer when computations must be carried out, and for some of our purposes we can do so by use of Monte Carlo simulation techniques.

The planner responsible for budgeting the transportation of recruits will wish to know not only the "best" way to transport them, but also what the effort is going to cost. A common measure of cost used in such situations as this, where there is uncertainty involved, is to determine the expected value of the cost; in this case it would be of minimum cost. Ideally the entire probability distribution of minimum cost is desired, but problems of this nature are often found to be too difficult to solve entirely. In this case even if we could solve for the expected value of minimum cost, we would not have arrived at

a very satisfactory answer to the question, "How cheaply can this transportation be carried out?" The reason the answer is unsatisfactory is that implicit in the use of the expected value of minimum cost to answer the question, is the assumption that a real system could be set up to handle any transportation input optimally. Instead of taking such an idealistic approach we have decided to be satisfied with trying to find a solution which will deviate least from this ideal. The "analytical" expression for the probability distribution of recruiting station inputs to the transportation problem (derived above) gives some idea of the complexity of an analytical approach to finding this expected value. In order to avoid the difficulty, the FORTRAN program for solving transportation problems was modified to solve a large number of randomized problems when given inputs of quotas, probability of interview success, and average rate of arrivals for each RS. Thus, it was possible to generate a vector of random inputs and to obtain a sample mean value of minimum costs. In order to give an indication of how the distribution of minimum costs varies the program was designed to solve for sample deviation also. Here again it should be noted that since many data were picked at random (i.e., the values of p_j , the probability of interview success, and α_j the average arrival rate for the j^{th} RS), the solution is only an illustration of method.

From the previously mentioned relation that

$$E[z_0(B)] \leq z_0(E[B]) \quad (4.1)$$

we see that a solution of the transportation problem using mean values of inputs will give us an upper bound on the set of possible means of optimal solutions. It is also one specific description of the input

situation for which we can find a minimum cost transportation pattern, but it may not be the "best".

What we seek for the "best" is first, a loss function describing the penalty we pay for improper action, and second, a rule for deciding what action to take so that we can expect to pay the least penalty. In other words, we are trying to solve a standard statistical decision problem and are trying to determine an estimator. Desirable properties of such estimators are discussed in [6] and other standard statistics texts. Those properties sometimes conflict and are often chosen somewhat arbitrarily. Our estimator will be chosen arbitrarily also.

Where we first define

$$\hat{z} = \min z(\hat{B})$$

for \hat{B} , an estimator of B , a natural loss function, λ , would seem to be

$$\lambda(\hat{B}; B) = \hat{z}(B; \hat{B}) - z_0(B)$$

where we write $\hat{z}(B; \hat{B})$ to emphasize the parametric role played by \hat{B} .

Later it will also be written without the \hat{B} .

The risk, or expected loss, we wish to minimize is

$$R(\hat{B}; B) = E [\lambda(\hat{B}; B)]$$

where \hat{B} represents the decision rule for arriving at the final action, the appropriate transformation on B .

Our rule, "use the estimator \hat{B} ", is that which consists of acting as though B were the actual value of B . The action then is

$$\hat{z} = C^T \hat{X}$$

$$\hat{z} = C^T \hat{P}^{-1} B$$

where we now regard \hat{z} as a transformation on \hat{B} . The vector \hat{X} , the "minimum basic feasible solution" is one of the results of the solution of our transportation problem under the restrictions represented by \hat{B} . Equation (4.1) becomes

$$\min R = \min_{\hat{B}} E \left[\hat{z}(B) - z_0(B) \right]$$

In words, equation (4.1) regards the loss as the difference between a fixed solution and an ideal solution. The fixed solution is that arrived at by use of the estimator \hat{B} . The ideal solution is that arrived at by use of the actual value of B .

Drawing on the notation of simplex algorithm one is tempted to say

$$\min R = \min_{\hat{B}} \left\{ C^T \hat{P}^{-1} (E[B]) - E \left[C^T \hat{P}^{-1} B \right] \right\}$$

making use of the linearity of the transformation represented by \hat{P}^{-1} , that transformation arrived at by a simplex solution using the estimator \hat{B} . In fact, the authors took this approach until certain computer outputs brought the inherent error (discussed below) to their attention. Continuing the above reasoning would have led to the following satisfying solution.

Letting $E[B] = \bar{B}$, and substituting, equation (a) becomes

$$E \left[C^T \hat{P}^{-1} B \right] = C^T \bar{P}^{-1} \bar{B}$$

where \bar{P}^{-1} is the transformation arrived at by use of $E[B]$ in the simplex

algorithm, and by the definition of

$$z_0(B) = \min \{z(B)\} = \min_{P^{-1}} \{C^T P^{-1} B\},$$

$$C^T \bar{P}^{-1} \bar{B} \leq C^T P^{-1} \bar{B}$$

for any other $P^{-1} \neq \bar{P}^{-1}$, therefore,

$$\min_{\hat{B}} R = \min_{\hat{B}} \{C^T \hat{P}^{-1} \bar{B} - E[C^T P^{-1} B]\}$$

$$\min R = \{C^T \bar{P}^{-1} \bar{B} - E[C^T P^{-1} B]\}$$

i.e., $\hat{B} = \bar{B}$

and $\hat{P}^{-1} = \bar{P}^{-1},$

the best policy is to use the mean value of B as an estimator and the transportation pattern computed from it.

The above approach is in error in that we assumed we could use the transformation \hat{P}^{-1} over the entire space of B vectors with some of the solutions becoming non-optimal, ignoring the possibility that they might become infeasible. However, the fact is that the solution to the linear programming problem is either optimal or else it is infeasible, where the term feasibility is technically defined. For a solution to be feasible it must satisfy the restrictions

$$AX = B$$

$$\text{and } x_i \geq 0 \quad \text{for all } i. \quad (4.2)$$

For the mathematical details of this aspect of the simplex algorithm, the reader is referred to Garvin [3].

We can restate the definition of feasibility by defining the elements of the P^{-1} matrix as p_{ik}^{-1} and writing for equation (4.2) (in the case where we started with \hat{B} and its implicit transformation \hat{P}^{-1})

$$\sum_{k=1}^m \hat{p}_{ik}^{-1} \beta_k < 0 ,$$

and we want to know the probability of this occurring, i.e.,

$$P \left[\sum_{k=1}^m p_{ik}^{-1} \beta_k < 0 \right]$$

The implication of the fact that $\hat{P}^{-1} B$ is optimal or else not feasible is that there are two regions in the space of B vectors and they are disjoint. In one region the loss function is zero-valued and \hat{P}^{-1} is optimal. In the other, the loss function takes on a positive value, so apparently the estimator \hat{B} that we seek is that one which is associated with the region of infeasibility with the least expectation. That is, the risk can be more accurately defined as

$$R(\hat{B}; B) = 0 + \int_{\{B \mid \hat{P}^{-1} B \notin K\}} \lambda(\hat{B}; B) dF_B$$

where K is the set of all feasible solutions to the equation $AX = B$. Thus, the estimator we seek is that which minimizes the second term on the right, i.e.,

$$\min_{\hat{B}} R(\hat{B}; B) = \min_{\hat{B}} \left\{ \int_{\{B \mid \hat{P}^{-1} B \notin K\}} \lambda(\hat{B}; B) dF_B \right\}$$

Now, defining $X^* = \hat{P}^{-1} B$, and x^* as some vector value,

$$\begin{aligned} \lambda(\hat{B}; B) &= 0 && \text{if } X^* \in K \\ &> 0 && \text{if } X^* \notin K, \text{ i.e., } X^* \in \bar{K} \end{aligned}$$

$$\begin{aligned} P[X^* \in K] &= \int_K dF_{X^*}(x^*) \\ P[X^* \notin K] &= \int_{\bar{K}} dF_{X^*}(x^*) \\ &= \int_{\bar{K}} f_{X^*}(x^*) dx^* \end{aligned}$$

where \bar{K} is the complement of K in the space of all $\hat{P}^{-1}B$,

and

$$\begin{aligned} f_{X^*}(x^*) &= f_{X_1, X_2, \dots, X_m}(x_1, \dots, x_m) \\ &= f_{\beta_1, \beta_2, \dots, \beta_m}(b_1, b_2, \dots, b_m) |J(b_1, \dots, b_m)|^{-1} \end{aligned}$$

Now

$$J(b_1, \dots, b_m) = \begin{vmatrix} \frac{\partial x_1}{\partial b_1} & \dots & \frac{\partial x_1}{\partial b_m} \\ \vdots & & \vdots \\ \frac{\partial x_m}{\partial b_1} & \dots & \frac{\partial x_m}{\partial b_m} \end{vmatrix} = |P^{-1}| \neq 0$$

therefore

$$f_{X^*}(x^*) = f_{\beta_1, \dots, \beta_m} \left(\sum_{i=1}^m p_{1i} x_i, \dots, \sum_{i=1}^m p_{mi} x_i \right) |P|$$

Here a difficulty arises due to the non-independence of β_{m+1} and β_i , $i = 1, \dots, m$.



However, assuming the foregoing operations have been carried out, we can make use of the central limit theorem to say that the random variables β_i , $i = 1, \dots, m$, can be said to be approximately normally distributed since they are large sums of random variables, at least over long periods of time. Further, since β_{m+1} is also a large sum of these sums, it can be approximated also as a normally distributed random variable which is independent of the other m . Consequently, we are able to write for these approximations

$$\begin{aligned} f_B(b) &= f_{\beta_1, \dots, \beta_m}(b_1, \dots, b_m) \\ &\cong \prod_{i=1}^m f_{\beta_i}(b_i) \end{aligned}$$

i.e., there is a density function which can serve our purpose.

Now

$$E [\lambda(\hat{B}; B)] = \int_{\{X^* | X^* \in K\}} \lambda(\hat{B}(\hat{X}); B(X^*)) dF_{X^*}.$$

To carry this out we must redefine \hat{z} to conform to reality, e.g.,

$$\hat{z}(B) = \sum_{i=1}^m c_i x_i^{*'}.$$

$$\begin{aligned} \text{where } x_i^{*'} &= 0 && \text{if } x_i^* < 0 \\ &= x_i^* + x^* && \text{if } x_{i-1}^* < 0 \text{ and } i \text{ is odd} \\ &= x_i^* + x_{i+1}^* && \text{if } x_{i+1}^* < 0 \text{ and } i \text{ is even} \\ &= x_i^* && \text{otherwise} \end{aligned}$$

(This definition of x_i^{*} models the action of an RS in carrying out a

fixed policy). Finally

$$E[\lambda] = \int_{-\infty}^0 \int_{-\infty}^0 \dots \int_{-\infty}^0 \left(\sum_{i=1}^m c_i x_i^* - \sum_{i=1}^m c_i x_{i0} \right) f_{\beta_1, \dots, \beta_m} \left(\sum_{i=1}^m p_{1i} x_i, \dots, \sum_{i=1}^m p_{mi} x_i \right) \left| P \right| dx_1, \dots, dx_m$$

Here again we have come to a point where for computational purposes, theory yields to simulation and the theory merely provides a conceptual framework for what we are trying to do. Of the ideas written above, at least the loss function can be useful in the simulation.

In investigating the results of the above reasoning by Monte Carlo simulation, a program was designed to provide the randomized input B vectors and solve for their sample mean (one common estimator). This mean vector was then used as an input. The program computed the results of transforming the randomized B vectors (using the transformation found in the solution for the mean value vector) altering it by using the definition given above for x_i^* outside the region of basic feasibility. It further computed the loss function in each case and the sample mean loss or sample risk for this estimator. In computing the random B vectors, the same concept of the events giving rise to enlistments was used as in the analytical description; however, because of the large numbers involved, and to speed computation time, a normal approximation to the Poisson distribution was used. For this approximation see Parzen [5], and for the formula used to compute random numbers according to a normal distribution see Vaa [9].

We are able to find sample risks given specific B estimators but we do not believe it is possible to solve for the best. The only one for which we have solved so far is the mean B vector, \bar{B} . For examples

of the results of this simulation (using completely arbitrary parameters) see Tables VII-A through VII-G, pages 55 to 61, and for a discussion of the results see Section 6.

TABLE VII-A

PROBLEM RECRUIT *SAMPLE*

NUMBER OF SAMPLES 15

ROUTE NUMBERS AND ASSOCIATED CCSTS

1	52.23	2	161.92	3	34.27	4	154.46	5	23.13	6	119.55
7	37.23	8	147.51	9	16.70	10	138.40	11	28.06	12	133.42
13	11.79	14	129.95	15	29.93	16	142.34	17	31.31	18	130.74
19	22.60	20	118.44	21	43.99	22	128.56	23	38.63	24	130.35
25	30.97	26	128.44	27	15.50	28	118.18	29	47.09	30	131.87
31	11.55	32	106.56	33	17.25	34	126.16	35	12.73	36	117.29
37	31.21	38	142.01	39	22.54	40	124.72	41	1.28	42	97.17
43	14.65	44	122.92	45	11.70	46	109.08	47	2.23	48	107.10
49	8.81	50	98.49	51	27.53	52	57.96	53	15.04	54	99.05
55	18.72	56	79.71	57	13.92	58	95.70	59	19.00	60	86.19
61	42.72	62	31.31	63	33.25	64	47.18	65	66.41	66	76.91
67	20.37	68	91.55	69	35.14	70	103.02	71	29.52	72	57.35
73	97.81	74	3.36	75	78.41	76	31.50	77	51.35	78	14.88
79	106.29	80	23.36								

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TABLE VII-B

NUMBER OF RECRUITING DAYS IN PERIOD 365

GREAT LAKES PERCENT .500

STATION NUMBER	QUOTA	AVERAGE INTERVIEW NUMBER	PROB OF INTERVIEW SUCCESS
1	2560.	10.	.550
2	4460.	18.	.650
3	2147.	19.	.500
4	7351.	42.	.700
5	1361.	6.	.650
6	1399.	6.	.500
7	1399.	6.	.650
8	1399.	6.	.700
9	1330.	3.	.750
10	2054.	5.	.600
11	1478.	6.	.750
12	1445.	12.	.700
13	1358.	9.	.750
14	2054.	6.	.750
15	1478.	6.	.700
16	1674.	6.	.750
17	2929.	12.	.750
18	1395.	12.	.650
19	4882.	18.	.800
20	3070.	19.	.600
21	4563.	15.	.500
22	4333.	15.	.600
23	1663.	18.	.550
24	1922.	8.	.600
25	2777.	12.	.550
26	1803.	12.	.650
27	1864.	9.	.600
28	2504.	9.	.700
29	2380.	15.	.750
30	1673.	18.	.700
31	1432.	6.	.700
32	1887.	9.	.750
33	2539.	12.	.700
34	1363.	6.	.700
35	1244.	6.	.450
36	1412.	6.	.600
37	6201.	27.	.700
38	2146.	9.	.650
39	4801.	21.	.700
40	2750.	12.	.750
TOTAL	9999.		



TABLE VII-C

SAMPLE 12

STATION NUMBER	NUMBER MEN ENLISTED	PERCENT CUCIA FILLED
1	1988.	77.656
2	4196.	94.081
3	1643.	76.525
4	7351.	100.000
5	1361.	100.000
6	1094.	78.199
7	1399.	100.000
8	1399.	100.000
9	1819.	98.675
10	1999.	96.835
11	1478.	100.000
12	1445.	100.000
13	1358.	100.000
14	2054.	100.000
15	1478.	100.000
16	1624.	97.013
17	2929.	100.000
18	1387.	97.276
19	4823.	100.000
20	1937.	100.000
21	3370.	64.267
22	3372.	71.738
23	1602.	96.392
24	1740.	90.531
25	2308.	85.848
26	1804.	100.000
27	1864.	100.000
28	2330.	100.000
29	1622.	100.000
30	1422.	100.000
31	1859.	100.000
32	1367.	100.000
33	1977.	78.537
34	1314.	93.059
35	6214.	100.000
36	2146.	100.000
37	4801.	100.000
38	2750.	100.000
39	4672.	93.467
40		
41		



TABLE VII-D

MINIMUM COST OF ALL TRANSPORTATION IS 4142953.99

ROUTES USED AND AMOUNTS

NUMBER MEN SHIPPED

ROUTES

1	1988.
3	4196.
5	1643.
7	7351.
9	1361.
11	1094.
13	1359.
15	1399.
17	1819.
19	1141.
21	1740.
23	3330.
25	1358.
27	2054.
29	1603.
31	1624.
33	1929.
35	1357.
37	4882.
39	1975.
41	1847.
43	1478.
45	1445.
47	1478.
49	1384.
51	1808.
53	1864.
55	2338.
57	3380.
59	1673.
61	1432.
63	1887.
65	2539.
67	1363.
69	1977.
71	1314.
73	6201.
75	2146.
77	4801.
79	2750.
81	3112.
83	

AVERAGE CCST

4159224.85

STD DEVIATION OF CCST

11172.45



TABLE VII-E

MEAN NUMBER ENLISTED

STATION NUMBER

1	2049.
2	42262.
3	1579.
4	7351.
5	1361.
6	1051.
7	1399.
8	1399.
9	1806.
10	1998.
11	1478.
12	2445.
13	1358.
14	2054.
15	1478.
16	1609.
17	2925.
18	1382.
19	1980.
20	3180.
21	3344.
22	1576.
23	1779.
24	2409.
25	1808.
26	1864.
27	2255.
28	2380.
29	1673.
30	1432.
31	1837.
32	2539.
33	1363.
34	1953.
35	1312.
36	6207.
37	2050.
38	4801.
39	2750.
40	46710.



TABLE VII-F

MINIMUM COST OF ALL TRANSPORTATION OF MEANS IS 4154715.82

ROUTES USED AND AMOUNTS
NUMBER MEN SHIPPED

ROUTES	NUMBER MEN SHIPPED
1	2049
3	4262
5	1579
7	7351
9	1361
11	1051
13	1399
15	1359
17	1306
19	971
21	1478
23	1180
25	1358
27	1354
29	1779
31	1606
33	2529
35	1382
37	4885
39	1927
41	1027
43	1476
45	1545
47	2409
49	2408
51	1864
53	1855
55	2380
57	3732
59	1432
61	1837
63	2339
65	1353
67	1312
69	1301
71	6250
73	2050
75	4801
77	2750
79	4234
81	4083
83	4083
85	4083
87	4083
89	4083
91	4083
93	4083
95	4083
97	4083
99	4083



SAMPLE	COST USING RANDOM INPUT	LCSS ON THIS SAMPLE
1	100	100
2	100	100
3	100	100
4	100	100
5	100	100
6	100	100
7	100	100
8	100	100
9	100	100
10	100	100
11	100	100
12	100	100
13	100	100
14	100	100
15	100	100
16	100	100
17	100	100
18	100	100
19	100	100
20	100	100
21	100	100
22	100	100
23	100	100
24	100	100
25	100	100
26	100	100
27	100	100
28	100	100
29	100	100
30	100	100
31	100	100
32	100	100
33	100	100
34	100	100
35	100	100
36	100	100
37	100	100
38	100	100
39	100	100
40	100	100
41	100	100
42	100	100
43	100	100
44	100	100
45	100	100
46	100	100
47	100	100
48	100	100
49	100	100
50	100	100
51	100	100
52	100	100
53	100	100
54	100	100
55	100	100
56	100	100
57	100	100
58	100	100
59	100	100
60	100	100
61	100	100
62	100	100
63	100	100
64	100	100
65	100	100
66	100	100
67	100	100
68	100	100
69	100	100
70	100	100
71	100	100
72	100	100
73	100	100
74	100	100
75	100	100
76	100	100
77	100	100
78	100	100
79	100	100
80	100	100
81	100	100
82	100	100
83	100	100
84	100	100
85	100	100
86	100	100
87	100	100
88	100	100
89	100	100
90	100	100
91	100	100
92	100	100
93	100	100
94	100	100
95	100	100
96	100	100
97	100	100
98	100	100
99	100	100
100	100	100

RISK	STD DEVIATION OF LOSS	STD DEVIATION OF LOSS
1	0.00	0.00
2	0.00	0.00
3	0.00	0.00
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00



APPENDIX C

OTHER POSSIBLE LINES OF INVESTIGATION

1. When a planner decides to seek a solution to a problem by using the criterion of minimum expected loss, he is considering all events (in our case B vector values) which have any positive probability of occurring. Suppose, however, he should prefer to make a wager based on the assumption that in the finite period of time his system (the U. S. Navy) is in operation, that events of extremely small probability will not occur. By basing his actions on such an assumption he is betting against nature that he can save an amount of money in this finite period of time, realizing that he risks losing more than the amount he saves, but risks it with a very low probability that the (extra) loss will occur. In effect, in our case he wishes to find the solution (B estimator) for the following objective function:

$$\min_{\hat{B}} \left\{ \int \lambda(Q; \hat{B}) \left[\hat{z}(B; \hat{B}) - z_0(B) \right] dF_L \right\}$$

where

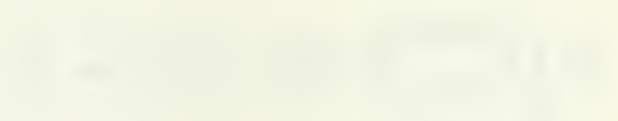
θ represents the upper limit of the small level of probability of the events he wishes to bet will not occur,

$\hat{z}(B; \hat{B})$ is the objective function solution of the transportation problem for \hat{B} evaluated at B and emphasizes the parametric role played by \hat{B} .

dF_L is the differential of the probability distribution function for $\lambda(B; \hat{B})$. $\lambda(B; \hat{B})_\theta$ is the θ^{th} percentile of $\lambda(B; \hat{B})$ under the probability law F_L .

That the bet mentioned above might be a desirable one to make

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could be determined in the following way. Suppose, as is likely, that the expected values of the β 's, the components of our random vector B , occur near their maximum values, the q 's. The reason this is likely is that the quotas of recruiting stations would probably be assigned in just such a manner. Also suppose \bar{B} is our best estimator using expectation. Further, suppose a sensitivity analysis showed that small changes of \bar{B} with values of β increasing should cause the solution to become non-optimal, i.e., the loss for small increments of β increases rapidly and should not satisfy the constraint that $x_i \geq 0$ for all x_i of the solution; and that equal small changes of \hat{B} with values of β decreasing should not cause the solution to become non-optimal, i.e., for small decrements of β , the loss remains at zero. The probability mass function of one of such β 's, $p_{\beta}^{(b)}$ would have a form similar to Figure A.

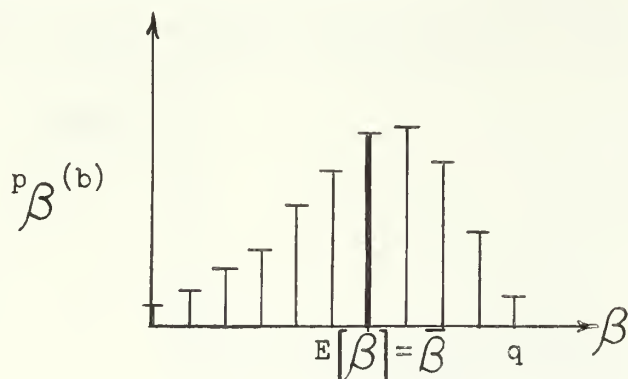


FIGURE A

From the figure we see that the probability that β takes on the exact value $(\bar{\beta} + \Delta\beta)$ is greater than the probability that it takes on the exact value $(\bar{\beta} - \Delta\beta)$

$$p_{\beta}(\bar{\beta} + \Delta\beta) > p_{\beta}(\bar{\beta} - \Delta\beta)$$

where $\Delta \beta$ is some arbitrary small change in β . This statement hinges on our assumption of the level of $\bar{\beta}$ relative to q and the fact that q is large, i.e., that $\bar{\beta}$ is on the increasing side of the mode of $p_{\beta}^{(b)}$ and so are both $(\bar{\beta} + \Delta \beta)$ and $(\bar{\beta} - \Delta \beta)$.

Looking in the same general way at the loss function, we see that it will probably have a form similar to that of Figure B,

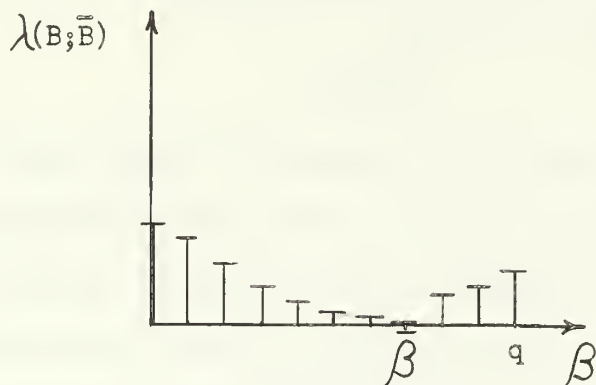


FIGURE B

since there will obviously be no loss at $\bar{\beta}$. It might (and a sensitivity analysis would determine this) look like Figure C,

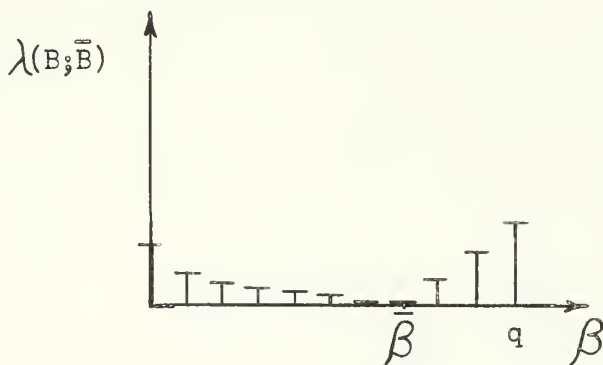


FIGURE C

so that, in the latter case the probability density function of $\lambda(B; \hat{\beta})$ might be similar to Figure D.

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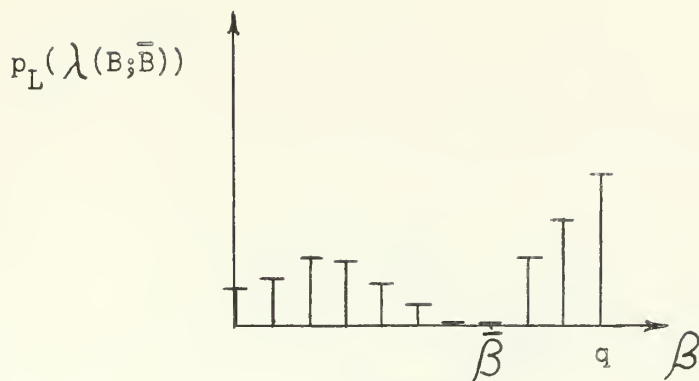


FIGURE D

Even so simple a discussion as that above makes it clear that, should these conditions hold, for all (or possibly even some smaller number) of the components of B , an estimator of B with components to the right of the prospective $\bar{\beta}$'s would be a reasonable choice, given the willingness to make the previously mentioned bet against nature. The choice of such a new estimator, \tilde{B} , might result in a loss function such as that in Figure E,

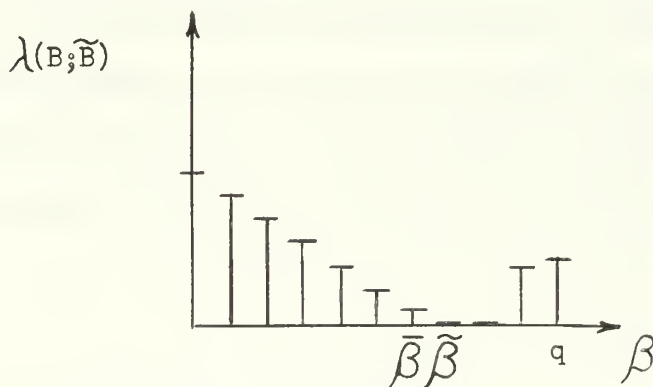


FIGURE E

and a probability mass function similar to that of Figure F.

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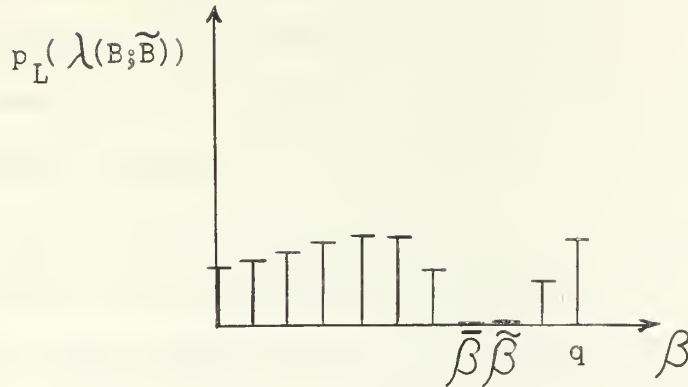


FIGURE F

The small standard deviations resulting from the parameters we used in our simulations, indicate that this second approach would not be profitable. However, were the numbers of recruits sufficiently small to "spread out" the probability mass functions more, an investigation of this additional approach might become profitable.

The effect of ignoring the tail, that is, those losses with probability less than θ , is illustrated in Figure G. In effect the new \hat{B} , \tilde{B} , can be found by renormalizing F_L truncated at the θ^{th} percentile and finding the new mean of the region of B vector which produce the truncated function. An approximation to this new mean (a new sample mean) could easily be found by use of a computer.

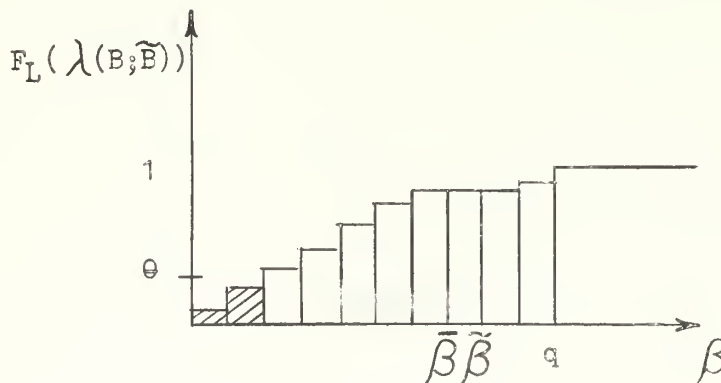


FIGURE G

2. A possible improvement that might be made in the solution of this problem of the transportation probability model would be obtained by investigating the problem's dynamic aspect. A more sophisticated model than ours might allow the planner to make an optimal decision at any point in the time period (one, n , or an "infinite" number of years) based upon the conditions then prevailing. It is believed that a multi-staged linear program such as is discussed by Dantzig [4] and [7], or a dynamic program approach such as those discussed by Bellman [8] could be used to provide this dynamic aspect.

APPENDIX D

GLOSSARY OF SYMBOLS

A	The matrix of coefficients of the system of linear equations represented by equations (A-2) and (A-2'). The matrix is defined in (A-3).
a_i	The total number of recruits to be shipped from the i^{th} RS, of which there are m .
α_j	The average number of candidates arriving per day at the j^{th} RS.
B	The adjoint column vector composed of the a_i 's and b_j 's.
\hat{B}	A fixed value estimator of B .
\bar{B}	The vector of expected values of the random variables β_j .
\tilde{B}	A (second) fixed value estimator of B .
β_j	The components of the vector B when the vector is considered as a vector of random variables.
b_j	A fixed value in the domain of β_j . Also the total number of recruits to be shipped to the j^{th} RTC, of which there are n .
c^T	The transposed column vector of c_{ij} 's.
c_{ij}	The total cost of shipping one recruit from the i^{th} RS to the j^{th} RTC.
γ	The proportion of total input assigned to the RTC of interest.
$E[V]$	The expected value of some random variable V in which V might be a scalar or vector. In the latter case, the symbol represents a vector of expected values.
F_v	The probability distribution function of some random variable V .
$J(\dots)$	The Jacobian determinant of a transformation with respect to the variables (\dots) .
K	The set of all feasible solutions to the linear program for $AX = \hat{B}$.
\bar{K}	The complement of K in the space of all possible $\hat{P}^{-1} B$ vectors.

$\lambda(B;\hat{B})$	A loss function, the difference between the least cost possible if the problem could be solved optimally for B, and the cost obtained using the transformation derived by using \hat{B} as the estimator.
N_j	The number of interviews occurring at the j^{th} RS.
P^{-1}	The inverse of the matrix composed of a set of linearly independent vectors, used as a transformation on B to yield X.
$\hat{P}^{-1}, \bar{P}^{-1}$	Those transformations arrived at by use of \hat{B} and \bar{B} respectively in simplex solutions of the transportation problem.
\hat{p}_{ik}^{-1}	The i, k^{th} element of matrix \hat{P}^{-1} .
$p_v(v)$	The probability mass function of some random variable V.
p_j	The probability that the j^{th} recruiter will enlist a candidate at an interview.
Q	The vector of q_j 's.
q_j	The assigned quota of recruits for the j^{th} RS.
$R(\hat{B};B)$	A regret function or expected value of loss.
r_j	$r_j = 1 - p_j$
θ	The level of probability of the events which the planner bets will not occur.
X	The column vector composed of the x_{ij} 's.
x_{ij}	The total number of recruits shipped from the i^{th} RS to the j^{th} RTC.
X_0	The vector of x_{ij} 's which yields the minimum z for the general case.
x_{i0}	An element of X_0 .
\hat{X}	The vector of x_{ij} 's which yields the minimum z when \hat{B} is the vector of restrictions.
X^*	The vector valued random variable resulting from the operation of the transformation \hat{P}^{-1} on the random vector B.
x^*	A possible value of X^* .
X_i^*	The i^{th} component of X^* .
x_i^*	The i^{th} component of x^* .

- x_i^* The transformed x_i^* which models the actual situation of an RS carrying out a policy.
- z The objective function expressed in equations (A-1) and (A-1').
- z_0 The transformation which yields the minimum objective function, i.e., $C^T X$.
- $z_0(B)$ The minimizing transformation found using B as a set of parameters and considering B as a vector of random variables.
- $\hat{z}(\hat{B})$ The minimizing transformation found by using \hat{B} as the specific value of B .
- $\hat{z}(B)$ The transformation of B which gives the minimum of the objective function when acting on \hat{B} .

1. The first part of the paper discusses the importance of the study of the history of the United States. It is argued that a knowledge of the past is essential for a full understanding of the present and for the development of a sound policy for the future.

2. The second part of the paper deals with the question of the rights of the individual. It is shown that the rights of the individual are not absolute and that they must be balanced against the needs of the community.

3. The third part of the paper discusses the role of the government in the life of the individual. It is argued that the government has a duty to protect the rights of the individual and to provide for the needs of the community.

4. The fourth part of the paper deals with the question of the future of the United States. It is argued that the future of the United States depends on the actions of the present generation and that it is the duty of the present generation to secure a bright and happy future for the United States.

APPENDIX E

```

C
C
C
PROGRAM LP2
A MODIFICATION OF LP1 TO GENERATE A TRANSPORTATION MATRIX
DIMENSION A(50,100),E(50),KB(100),X(50),JH(50),P(50)
1 DIMENSION Y(50),E(50,50),Z(100),DDT(100),PINV(50,50),XI(50)
DIMENSION INFIX(10),TOL(10),KOUT(10),ERR(10),RUN(8)
EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
ICASE=1
2 GO TO (992,993),ICASE
992 DO 10 I=1,100
DO 10 J=1,50
10 A(I,J)=0.
C
C
C
PERMANENT DATA
INFIX(3)=MAX NO. OF ROWS (CF. DIMENSION STATEMENT)
INFIX(7)=MAX INTERATION COUNT
B(1)=0.
INFIX(1)=4
INFIX(3)=50
INFIX(5)=2
INFIX(6)=1
INFIX(7)=100
INFIX(8)=0
TOL(1)=1.E-7
TOL(2)=1.E-5
TOL(3)=-1.E-6
TOL(4)=1.E-7
PRM=0.
C
C
C
INPUT -- NOTE.. MROW = NO. OF ROWS PLUS ONE, BECAUSE COST
COEFFICIENTS ARE ENTERED AS ROW ONE.
C
C
C
ICASE=2
READ INPUT TAPE 2,3,NCOL,MROW,RUN
FORMAT (2I5,7A8,A6)
3 IF (NRUN-4HSTOP)9999,9998,9999
9998 WRITE OUTPUT TAPE 3,9997
9997 FORMAT (1H1,10X,12HJOB COMPLETE)
STOP
9999 NCOL=NCOL*MRWW
8 IF (NCOL) 500,500,8
MRROW=NCOL+MRWW
MRWW1=MRWW+1
MRWW2=MRWW+2
5 READ INPUT TAPE 2,4,(A(1,J),J=1,NCOL)
C

```

```

LP2000010
LP2000011
LP2000012
LP2000011
LP1000020
LP1000010
LP1000050
LP1000050
COA1
COA2
COA3
LP1000070
LP1000080
LP1000090
LP1000100
LP1000100
LP1000120
LP1000130
LP1000140
LP1000150
LP1000160
LP1000170
LP1000180
LP1000190
LP1000200
LP1000210
LP1000220
LP1000230
LP1000240
LP1000250
LP1000260
LP1000270
LP1000280
LP1000290
COA4
LP200300
LP100310
LP200301
LP200302
LP200303
LP200304
LP200305
LP100320
LP200325
LP200330
LP200335
LP200340
LP200345

```



```

C          NOTE- THE A(I,J) ROW ARE THE COST COEFFICIENTS
C          THE C-I,J-S ARE READ IN ROW-WISE
C
      4  FORMAT (7F10.5)
         J=0
         DO 4000 I=2,MRWW1
           DO 4000 K=1,NCLL
             J=J+1
           A(I,J)=1.0
         4000
         J=0
         DO 4001 I=MRWW2,MROW
           J=J+1
           DO 4001 K=J,NCOL,NCLL
             A(I,K)=1.0
           MROW2=MROW+2
           READ INPUT TAPE 2,4,(B(I),I=2,MROW2)
           B(MROW)=B(MROW)-B(MROW2)
         993  B(MROW)=B(MROW)+B(MROW2)
           IF(B(MROW)-B(MROW+1))994,994,992
         994  CONTINUE
C
C ***** NOTE-THE ORIGIN INFORMATION IS READ FIRST, FOLLOWED BY THE
C          DESTINATION INFORMATION. THE LAST DESTINATION IS EXCLUDED.
C
      29  WRITE OUTPUT TAPE 3,29, RUN
           FORMAT (1H1,28X,7A8,A6)
           WRITE OUTPUT TAPE 3,100, (A(1,J)J=1,NCOL)
      100  FORMAT (///10X,17HCOST COEFFICIENTS// (10X,8E13.6))
           DO 105 I=2,MROW2
             K=I-1
           WRITE OUTPUT TAPE 3,102,K,(A(I,J)J=1,NCOL)
           FORMAT (//3X,4HRDW 12,1X,8E13.6/(10X,8E13.6))
      105  WRITE OUTPUT TAPE 3,28,K,B(I)
           DO 28  FORMAT (5H  B(I2(1H),2X,E13.6)
C
C          CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DDT)
C
           WRITE OUTPUT TAPE 3,29,RUN
           WRITE OUTPUT TAPE 3,30,Y(1),B(MROW)
      30  FORMAT (///34X38HMINIMUM COST OF OBJECTIVE FUNCTION IS E13.6////
           145X,20HGREAT LAKES PERCENT E13.3////
           145X,30HBASIS VECTORS AND COEFFICIENTS///37X,6HVECTOR,10X,30HCoeffi
           2CIENT (X-ZERO COMPONENT)//)
C
C          FOLLOWING IS SET UP FOR ADDITIONAL OUTPUT IF REQUIRED
C
           DO 33 I=1,NCOL
             II=KB(I)

```

```

LP100350
LP200355
LP200356
LP100360
LP200361
LP200362
LP200363
LP200364
LP200365
LP200366
LP200367
LP200368
LP200369
LP200370
COA5
LP200372
COA6
COA7
COA8
COA10
LP200373
LP200374
LP200375
LP200376
LP100380
LP100390
LP100500
LP100510
LP100520
LP100530
LP100540
LP100560
LP100570
LP100580
LP100590
LP100600
LP100610
LP100620
LP100630
LP100640
LP 641
LP100650
LP100650
LP100660
LP100670
LP100680
LP100990
LP100700

```



```

31 IF(I1) 31,31,32
31 Z(I)=0.
32 GO TO 33
33 Z(I)=X(I1)
33 CONTINUE
35 NDE=7
35 IF (NDE-NCOL) 35,36,36
36 NDE=NCOL
37 IF (NDE-MROW) 37,40,40
37 NDE=MROW
C
C BACK TO NORMAL
C
40 WRITE OUTPUT TAPE 3,43,(JH(I),X(I),I=2,MROW)
43 FORMAT (37X,2HP(I2,1H),19X,E13.6)
C
C7733 WRITE OUTPUT TAPE 3,7733
7733 FORMAT(1H1,47X,23HNEGATIVE OF Z(J) - C(J)//48X,1HJ,10X,12H-(Z(J)-
1C(J))//)
DO 60 I=1, NCOL
DO 50 J=2,MROW
IF (I-JH(J)) 50,45,50
50 CONTINUE
C7734 WRITE OUTPUT TAPE 3,7734,I,DDI(I)
7734 FORMAT (48X,I2(9X,E13.6)
GO TO 60
CONTINUE
C7735 WRITE OUTPUT TAPE 3,7735,I,DDI(I)
7735 FORMAT (48X,I2,9X,E13.6,15H (BASIS VECTOR))
C
C K=MROW*MROW
DO 1001 I=1,MROW
L=0
DO 1001 J=1,K,MROW
L=L+1
PINV(I,L)=E(J)
1001 WRITE OUTPUT TAPE 3,2000
2000 FORMAT (1H1,52X,14HINVERSE MATRIX// )
C1010 DO 1010 I=2,MROW
2001 WRITE OUTPUT TAPE 3,2001,(PINV(I,J),J=2,MROW)
C1010 FORMAT (1H0/(6E13.6))
C
CALL ERROR(KOUT(1))
GO TO 2
500 CALL ERROR (7)
GO TO 2
END

```

LP100710
LP100720
LP100730
LP100740
LP100750
LP100760
LP100770
LP100780
LP100790
LP100800
LP100810
LP100820
LP100830
LP100840
LP100850
LP100860
LP100870
LP100880
LP100890
LP100900
LP100910
LP100920
LP100930
LP100940
LP100950
LP100960

LP100970
LP100980
LP100990
LP101000
LP101010
LP101020
LP101030
LP101040
LP101050
LP101060
LP101070
LP101080
LP101090
LP201095
LP101110
LP101200
LP101210
LP101220
LP101230
LP101240
LP101250



ERR00010
 ERR00020
 ERR00030
 ERR00040
 ERR00050
 ERR00060
 ERR00070
 ERR00080
 ERR00090
 ERR00100
 ERR00110
 ERR00120
 ERR00130
 ERR00140
 ERR00150
 ERR00160
 ERR00170
 MSU00920
 MSU00930
 MSU00940
 MSU00946
 MSU00950
 MSU00960
 MSU00970
 MSU00980
 MSU00990
 MSU01000
 MSU01010
 MSU01020
 MSU01030
 MSU01040
 MSU01050
 01060
 01070
 01080
 MSU01090
 MSU01100
 MSU01110
 MSU01120
 MSU01130
 MSU01140
 MSU01150
 MSU01160
 MSU01170
 MSU01180
 MSU01190
 MSU01200
 MSU01210

```

SUBROUTINE ERROR(KK)
  IF (KK-4) 2,70,62
  IF (KK-7) 63,72,2
  70 WRITE OUTPUT TAPE 3,71
  71 FORMAT (21HONO FEASIBLE SOLUTION)
  GO TO 2
  63 IF (KK-5) 2,64,67
  64 WRITE OUTPUT TAPE 3,65
  65 FORMAT (28HONO PIVOT, INFINITE SOLUTION)
  GO TO 2
  67 WRITE OUTPUT TAPE 3,68
  68 FORMAT (26HON ITERATION LIMIT EXCEEDED)
  GO TO 2
  72 WRITE OUTPUT TAPE 3,73
  73 FORMAT (23HON ILLEGAL INPUT QUANTITY)
  RETURN
  2
END
SUBROUTINE SIMPLX (INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DDT)
  DIMENSION INFIX(3),A(1),B(1),TOL(4),KOUT(7),ERR(8),JH(1),X(1),
  1 P(1),Y(1),KB(1),E(1),ZZ(3),IOFIX(16),TERR(8)
  1,DDT(1)
  EQUIVALENCE (INFLAG,IOFIX(1)),(NZ,IOFIX(2)),
  1 (MC,IOFIX(6)),(ME,IOFIX(3)),(MZ,IOFIX(4)),(MF,IOFIX(5)),
  2 (KZ,IOFIX(9)),(NCUT,IOFIX(7)),(NVER,IOFIX(8)),
  3 (NUMVR,IOFIX(12)),(ITER,IOFIX(10)),(INVC,IOFIX(11)),
  4 (INFS,IOFIX(14)),(NUNPV,IOFIX(13)),
  5 (ZZ(1),TPIV), (ZZ(2),ZERO), (ZZ(3),TCOST)
  6 DO 1340 I=1,8
  7 TERR(I)=0.0
  8 IOFIX(I+8)=INFIX(I)
  1340 IOFIX(I)=INFIX(I)
  N=NZ
  M=MZ
  K=KZ
  LA=1308
  DO 1308 I=1,3
  1308 ZZ(I)=TOL(I)
  TCOST=-ABS(TCOST)
  PMIX=PRM
  M2=M*M
  INFS=1
  IF (N)=1304,1371,1372
  IF (M-MF)=1304,1304,1372
  IF (MF-MC)=1304,1304,1373
  IF (MC-MC)=1304,1304,1374
  IF (ME-M)=1304,1375,1375
  IF (XMODE(INFLAG,4))=1,1400,1320,100
  
```


MSU01220
MSU01230
MSU01240
MSU01250
MSU01260
MSU01270
MSU01280
MSU01290
MSU01300
MSU01310
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MSU01330
MSU01340
MSU01350
MSU01360
MSU01370
MSU01380
MSU01390
MSU01400
MSU01410
MSU01420
MSU01430
MSU01440
MSU01450
MSU01460
MSU01470
MSU01480
MSU01490
MSU01500
MSU01510
MSU01520
MSU01530
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MSU01560
MSU01570
MSU01580
MSU01590
MSU01600
MSU01610
MSU01620
MSU01630
MSU01640
MSU01650
MSU01660
MSU01670
MSU01680
MSU01690

```

1400 DO 1401 I = 1, M
1401 JH(I) = 0
      KT = 0
      DO 1402 J = 1, N
      KB(J) = 0
      MM = KT + MF
      LL = KT + M
      KQ = 0
      DO 1403 L = MM, LL
      IF (A(L)) 1404, 1403, 1404
1404 KQ = KQ + 1
      LQ = L
      CONTINUE
1403 IF (KQ - 1) 1402, 1405, 1402
1405 IA = LQ - KT
      IF ( JH(IA) ) 1402, 1406, 1402
1406 IF (A(LQ)*B(IA)) 1402, 1407, 1407
1407 JH(IA) = J
      KB(J) = IA
      KT = KT + ME
1402 CONTINUE
1320 ASSIGN 1102 TO KPIV
1100 ASSIGN 1114 TO KJMY
      IF (LA) 1121, 1121, 1122
1121 INVC = 0
1122 NUMVR = NUMVR + 1
1101 DO 1101 I = 1, M2
      E(I) = 0.
      MM = 1
      DO 1113 I = 1, M
      E(MM) = 1.0
      X(I) = B(I)
1113 MM = MM + 1
      DO 1110 I = ME, M
      IF (JH(I)) 1111, 1110, 1111
1111 JH(I) = 12345
1110 CONTINUE
      INFS = 1
      DO 1102 JT = 1, N
      IF ( KB(JT) ) 600, 1102, 600
1114 TY = 0
      DO 1104 I = ME, M
      IF (JH(I) - 12345) 1104, 1105, 1104
1105 IF (.ABSF( Y(I) ) - TY) 1104, 1104, 1106
1106 IR = I
      TY = ABSF ( Y(I) )
1104 CONTINUE
      IF (TY - TPIV) 1107, 1108, 1108

```




MSU01700
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MSU01830
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MSU01880
MSU01890
MSU01900
MSU01910
MSU01920
MSU01930
MSU01940
MSU01950
MSU01960
MSU01970
MSU01980
MSU01990
MSU02000
MSU02010
MSU02020
MSU02030
MSU02040
MSU02050
MSU02060
MSU02070
MSU02080
MSU02090
MSU02100
MSU02110
MSU02120
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MSU02160
MSU02170

```

1107 KB(JT)=0
      GO TO 1102
1108 JH(IR)=JT
      KE(JT)=IR
      GO TO 900
1102 DO 1109 I=1,M
      CONTINUE
1112 IF (JH(I)-12345) 1109, 1112, 1109
1109 JH(I)=0
1100 CONTINUE
      ASSIGN 705 TO NDEL
      ASSIGN 1000 TO KJMY
      ASSIGN 221 TO KPIV
      JIN=0
1200 NEG=0
      DO 1201 I=MF,M
      CONTINUE
1202 IF (ABSF(X(I))-TZERO) 1202, 1203, 1203
      X(I)=0.0
1203 GO TO 1201
1205 IF (X(I)) 1203, 1201, 1205
1208 IF (JH(I)) 1201, 1206, 1201
1206 NEG=1
      JIN=1
1201 CONTINUE
      IF (INFS-JIN) 1320, 500, 200
200 INFS=0
201 PMIX=0.0
500 DO 503 J=1,M
502 P(J)=E(MM)
503 MM=MM+M
      IF (INFS) 501, 599, 501
501 DO 504 J=1,M
504 P(J)=P(J)*PMIX
      DO 505 I=MF,M
      MM=I
      IF (X(I)) 506, 507, 507
506 DO 508 J=1,M
507 P(J)=P(J)+E(MM)
508 MM=MM+M
      GO TO 505
507 IF (JH(I)) 505, 509, 505
509 DO 510 J=1,M
510 P(J)=P(J)-E(MM)
      MM=MM+M
505 CONTINUE
599 CONTINUE
700 JT=0

```



MSU02180
MSU02190
MSU02195
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MSU02640

```

701 BB = TCOST
702 DD = 702 JM = 1, N
703 IF ( ( KB(JM) ) ) 702, 300, 702
705 IF ( ( DT - BB ) ) 708, 702, 702
708 BB = DT
JT = JM
702 CONTINUE
IF (JT) 203, 203, 600
203 K = 3 + INFS
KZ = K
GO TO 257
600 DD 610 I = 1, M
610 Y(I) = 0.
JT*ME - ME
LL = 0
DD 605 I = 1, M
LP = LP + 1
IF (A(LP)) 601, 602, 601
601 DD 606 J = 1, M
LL = LL + 1
LL Y(J) = Y(J) + A(LP) * E(LL)
606 GO TO 605
602 LL = LL + M
605 CONTINUE
699 GO TO KJMY, ( 1000, 1114, 1392 )
1000 AA = 0.0
IA = 0.0
DD 1050 I = MF, M
IF ( X(I) ) 1050, 1041, 1050
YI = ABSF ( Y(I) )
YI - TPIV ) 1050, 1050, 1042
1042 IF ( ( JH(I) ) ) 1043, 1044, 1043
1043 IF ( ( IA ) ) 1050, 1048, 1050
1048 IF ( ( Y(I) ) ) 1050, 1050, 1045
1044 IF ( ( IA ) ) 1045, 1046, 1045
1045 IF ( ( YI ) ) 1050, 1050, 1047
1046 AA = 1
1047 AA = YI
IR = YI
1050 CONTINUE
IF (IR) 1099, 1001, 1099
1001 AA = 1.0E+20
DD 1010 IT = MF, M
IF ( ( Y(IT) ) ) 1010, 1010, 1002
1002 IF ( ( X(IT) ) ) 1010, 1010, 1003
1003 XY = X(IT) / Y(IT)

```


MSU02650
MSU02660
MSU02670
MSU02680
MSU02690
MSU02700
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MSU02730
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MSU02770
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MSU02800
MSU02810
MSU02820
MSU02830
MSU02840
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MSU02860
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MSU02900
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MSU02930
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MSU02960
MSU02970
MSU02980
MSU02990
MSU03000
MSU03010
MSU03020
MSU03030
MSU03040
MSU03050
MSU03060
MSU03070
MSU03080
MSU03090
MSU03100
MSU03110
MSU03120

```

1005 IF ( XY - AA ) 1004, 1005, 1010
1004 IF ( JH(IT)) 1010, 1004, 1010
1010 AA = XY
1010 IR = IT
1016 CONTINUE
1016 IF (NEG) 1016, 1099, 1016
1016 BB = -PIV
1016 DO 1030 I = MF, M
1012 IF (X(I)) 1012, 1030, 1030
1022 IF ( ( Y(I) ) 1022, 1030, 1030
1024 IF ( ( Y(I) * AA - X(I) )
1024 BB = Y(I)
1030 IR = I
1030 CONTINUE
1099 CONTINUE
206 IF( IR ) 207, 207, 210
207 KZ = K
257 IF (PMIX) 201, 400, 201
210 IF (ITER -NCUT) 900, 160, 160
900 NUMPV = NUMPV + 1
YI = -Y(IR)
Y(IR) = -1.
LL = 0
DO 904 L = IR, M2, M
903 IF ( E(L) ) 905, 914, 905
914 LL = LL + M
GO TO 904
905 XY = E(L) / YI
E(L) = 0.
DO 906 I = 1, M
LL = LL + 1
E(LL) = E(LL) + XY * Y(I)
906 CONTINUE
904 XY = X(IR) / YI
X(IR) = 0.
DO 908 I = 1, M
908 X(I) = X(I) + XY * Y(I)
Y(IR) = -YI
GO TO KPIV, ( 221, 1102 )
221 IA = JH(IR)
214 IF ( IA ) = 0
213 KB(IA) = IR
KB(JT) = JT
JH(IR) = 0
LA = 0
ITER = ITER + 1
INVC = INVC + 1

```



```

160 IF ( INVC - NVER ) 1200, 1320, 1200
KZ = 6
KZ = K
400 ASSIGN 410 TO NJEL
DO 401 I = 1, M
401 Y(I) = -B(I)
DO 402 I = 1, M
JA = JH(I)
IF (JA) 403, 402, 403
403 IA = ME* (JA-1)
DO 405 II = 1, M
IA = IA + 1
IF (A(IA) ) 415, 405, 415
415 Y(II) = Y(II) + X(I) * A(IA)
405 CONTINUE
402 CONTINUE
DO 481 I = 1, M
YI = Y(I)
IF ( JH(I) ) 472, 471, 472
471 YI = YI + X(I)
472 TERR(LA+1) = TERR(LA+1) + ABSF(YI)
482 IF ( ABSF (TERR(LA+2)) - ABSF ( YI ) ) 482, 481, 481
481 TERR(LA+2) = YI
CONTINUE
DO 411 I = 1, M
JM = JH(I)
IF ( JM ) 303, 411, 300
410 TERR(LA+3) = TERR(LA+3) + ABSF(DT)
IF ( ABSF(TERR(LA+4)) - ABSF(DT) ) 413, 411, 411
413 IF ( ABSF(TERR(LA+4)) - DT
411 TERR(LA+4) = DT
CONTINUE
IF (LA) 193, 191, 193
191 LA = 4
IF ( INFLAG - 4 ) 1320, 193, 193
193 IF (K-5) 1392, 194,
194 ASSIGN 1392 TO KJMY
GO TO 600
1304 KZ = K
1392 KZ = K 1309 I = 1, 8
1309 ERR(I) = 1, 8 TERR(I)
DO 1329 I = 1, 7
1329 KOUT(I) = IOFK(I+8)
300 RETURN
DT = 0.
LL = (JM - 1) * ME
301 DO 303 MM = 1, M
LL = LL + 1

```

```

MSU03130
MSU03140
03150
MSU03160
MSU03170
MSU03180
MSU03190
MSU03200
MSU03210
MSU03220
MSU03230
MSU03240
MSU03250
MSU03260
MSU03270
MSU03280
MSU03290
MSU03300
MSU03310
MSU03320
MSU03330
MSU03340
MSU03350
MSU03360
MSU03370
MSU03380
MSU03390
MSU03400
MSU03410
MSU03420
MSU03430
MSU03440
MSU03450
MSU03460
MSU03470
MSU03480
MSU03490
MSU03500
03510
MSU03520
MSU03530
MSU03540
MSU03550
MSU03560
MSU03570
MSU03580
MSU03590
MSU03600

```



```

304 IF ( A( LL )) 304, 303, 304 )
303 DT = DT + P( MM ) * A( LL )
399 CONTINUE
      DDT(JM)=DT
      CO TO NDEL , ( 410 , 705 )
      END
      END

```

```

MSU03610
MSU03620
MSU03630
MSU03635
MSU03640
MSU03650
MSU03660

```


APPENDIX F

```

PROGRAM LP2
DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50),Y(50),
1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
2KOUT(10),ERR(10),RUN(8),QUOTA(50),PRJB(50),AV(50),ANS(50),
3BSTORE(50,52),CLJSS(50),ZBAR(50),PERCT(50)
OCCOMMON TOL,R,Y,E,Z,CDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
1INCLL,MRWW,MROW,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
2CDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLOSS,VARCST
EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))

```

LP200012
LP200011

A MODIFICATION OF LP1 TO GENERATE A TRANSPCRATIION MATRIX

```

2 DO 10 I=1,100
DO 10 J=1,50
10 A(I,J)=0.

```

LP100100

PERMANENT DATA
INFIX(3)=MAX NO. OF ROWS (CF. DIMENSION STATEMENT)
INFIX(7)=MAX INTERATION COUNT

```

B(1)=C.
INFIX(1)=4
INFIX(3)=50
INFIX(5)=2
INFIX(6)=1
INFIX(7)=100
INFIX(8)=0
TOL(1)=1.E-7
TOL(2)=1.E-5
TOL(3)=-1.E-6
TOL(4)=1.E-7
PRM=0.

```

INPUT -- NOTE.. MROW = NO. OF ROWS PLUS ONE, BECAUSE COST
COEFFICIENTS ARE ENTERED AS ROW CNE.

```

READ INPUT TAPE 2,3,NCOL,MRWW,RUN
FORMAT (2I5,7A8,A6)
IF (NRUN-4HSTOP)9999,9998,9999
WRITE OUTPUT TAPE 3,9997
FORMAT (1H1,10X,12HJOB COMPLETE)
STOP

```

```

9999 NCOL=ACLL*MRWW
IF (NCOL).500,500,8
8 MROW=ACLL+MRWW
MRWW1=MRWW+1
MRWW2=MRWW+2

```

LP100190
LP100200
LP100210
LP100220
LP100230
LP100240
LP100250
LP100260
LP100270
LP100280
LP100290
LP200300
LP200310
LP200301
LP200302
LP200303
LP200304
LP200305
LP200320
LP200325
LP200330
LP200335

LP200340
LP200345
LP200350
LP100355
LP200355
LP200356

LP100360
LP200361
LP200362
LP200363
LP200364
LP200365
LP200366
LP200367
LP200368
LP200369
LP200370

PROB 103
LP100620

PROB 6
TEST 70
PROB 107
TEST 720

```

5 READ INPUT TAPE 2,4,(A(1,J),J=1,NCOL)
C
MROW1= MROW-1
C      NOTE- THE A(1,J) ROW ARE THE COST CCEFFICIENTS
C      THE C-I,J-S ARE READ IN ROW-WISE
C
7733 READ INPUT TAPE 2,7733,MAX,PRCP,TIME
4      FORMAT (110,2F10.5)
7734      FORMAT (7F10.5)
C      J=0
C      DO 4000 I=2,MROW1
C      DO 4000 K=1,NCCL
C      J=J+1
C      A(I,J)=1.0
4000      J=0
C      DO 4001 I=MROW2,MROW
C      J=J+1
C      DO 4001 K=J,NCCL,NCCL
C      A(I,K)=1.0
4001      A(I,K)=1.0
C
MAX1=MAX+1
AMAX=MAX
C      DO 8000 N=1,MAX2
C      DO 8000 J=2,MROW
8000      BSTORE(J,M)=0.0
C      VARCST=0.0
C      SUMANS=0.0
C      WRITE OUTPUT TAPE 3,29,RUN
29      FORMAT(1H1,36X,7A8,A6)
C      ITIME=XINTF(TIME)
C      WRITE OUTPUT TAPE 3,7734,MAX
7734      FORMAT(///,48X,17HNUMBER OF SAMPLES,I5)
C      WRITE OUTPUT TAPE 3,7770
7770      FORMAT(///,43X,34HROUTE NUMBERS AND ASSOCIATED CCSTS ///)
C      WRITE OUTPUT TAPE 3,7774,((J,A(1,J)),J=1,NCCL)
7774      FORMAT(13X,6(13,F8.2,5X))
C      READ INPUT TAPE 2,7,QUOTA(1),I=2,MROW1
7      READ INPUT TAPE 2,7,QUOTA(1),I=2,MROW1
C      READ INPUT TAPE 2,9,(PROB(I),AV(I),I=2,MROW1)
9      READ INPUT TAPE 2,9,(PROB(I),AV(I),I=2,MROW1)
C      WRITE OUTPUT TAPE 3,7775,ITIME,PROP
7775      FORMAT(1H1,40X,35HNUMBER OF RECRUITING DAYS IN PERIOD,I5 ///)
C      143X,15HGREAT LAKES PERCENT,3X,F10.3)
7735      WRITE OUTPUT TAPE 3,7735
C      FORMAT(///,17X,14HSTATION NUMBER,7X,24HAVERAGE INTERVIEW
1      NUMBER,5X,25HPROB OF INTERVIEW SUCCESS//)
C      DO 7736 J=2,MROW1
C      I=J-1

```



```

7736 WRITE OUTPUT TAPE 3,7736,I,QUOTA(J),AV(J),PROB(J)
      FORMAT(23X,I2,5X,F13.0,9X,F13.0,17X,F10.3)
      TOTAL=0.C
      DO 7737 J=2,MROW1
7737   TOTAL=TOTAL+QUOTA(J)
      PRINT 7738,TOTAL
7738   FORMAT(/25X,SHTOTAL,F13.0)
      DO 6666 N=1,MAX
      CALL RCRUIT(M)
      C ***** NCTE-THE ORIGIN INFORMATION IS READ FIRST, FOLLOWED BY THE
      C       DESTINATION INFORMATION. THE LAST DESTINATION IS EXCLUDED.
      C
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DCT)
      C
      WRITE OUTPUT TAPE 3,30,Y(1)
      FORMAT(11F1,///29X,37HMINIMUM COST OF ALL TRANSPORTATION IS 2X,F13.
30 12,///43X,23HROUTES USED AND AMOUNTS ///36X,7HROUTES ,16X,18HNUMBE
      2R MEN SHIPPED //)
      C
      WRITE OUTPUT TAPE 3,43,(JH(1),X(1),I=2,MROW)
43 19X,F13.0)
      C
      K=MROW*MROW
      DO 1001 I=1,MROW
      L=0
      DO 1001 J=I,K,MROW
      L=L+1
1001   PINV(I,L)=E(J)
      CALL ERCCR(KOUT(I))
      GO TO 8325
500   CALL ERCCR (7)
      GO TO 8325
8325   CALL AVERAG(M)
6666   CONTINUE
      CALL RISK
      END
      C
      SUBROUTINE ERROR(KK)
      IF (KK-4) 2,70,62
      IF (KK-7) 63,72,2
72 70 WRITE OUTPUT TAPE 3,71
71 71 FORMAT (21HNO FEASIBLE SOLUTION)
      GO TO 2
63 63 IF (KK-5) 2,64,67
64 64 WRITE OUTPUT TAPE 3,65
65 65 FORMAT (28HNO PIVOT, INFINITE SOLUTION)
      GO TO 2
      LP100830
      LP100840
      LP101000
      LP101010
      LP101020
      LP101030
      LP101040
      LP101050
      LP101060
      LP101210
      LP101220
      LP101230
      LP101240
      LP101250
      LP100660
      ERR00010
      ERR00020
      ERR00030
      ERR00040
      ERR00050
      ERR00060
      ERR00070
      ERR00080
      ERR00090
      ERR00100

```


ERR000110
ERR000120
ERR000130
ERR000140
ERR000150
ERR000160
M:SU03650

67 WRITE OUTPUT TAPE 3,68
68 FORMAT (26H) ITERATION LIMIT EXCEEDED)
GO TO 2
72 WRITE OUTPUT TAPE 3,73
73 FORMAT(23H) ILLEGAL INPUT QUANTITY)
2 RETURN
END

C

SUBROUTINE RCRUIT (MIKE)
OCIMENSION A(50,100), B(50), KR(100), X(50), JH(50), P(50), Y(50),
1E(50,50), Z(100), DT(100), PINV(50,50), XI(50), INFIX(10), TOL(10),
2KOUT(10), ERR(10), RUN(8), QUITA(50), PROB(50), AV(50), ANS(50),
3BSTORE(50,52), CLOSS(50), ZBAR(50), PERCT(50)
OCOMMON TOL, R, Y, E, Z, DDT, PINV, XI, A, B, K3, X, JH, P, INFIX, KOUT, ERR, RUN,
1NCLL, MRWW, MROW, MAX, TIME, PROP, SUMANS, BSTORE, AMAX, MAX1, MAX2, PRM,
2DDT, NCOL, MROW, PERCT, QUITA, PROB, AV, ANS, BSTORE, CLOSS, VARCST
EQUIVALENCE (INFX(2), NCOL), (INFX(4), MROW), (NRUN, RUN(1))
WRITE OUTPUT TAPE 3,7776, MIKE
FORMAT(1H1, //45X, 6HSAMPLE, I4, //20X, 14H) STATION NUMBER , 6X, 19H) NUMBE
7776 1R MEN ENLISTED , 5X, 20H) PERCENT QUITA FILLED ///

TEST 150
PROB 16
PROB 130
PROB 230
PROB 26
PROB 27
PROB 28

PROB030
PROB0031

PROB0033

PROB 340
TEST1330

TEST 340
PROB0035
PROB0036
PROB0037
PROB0038
PROB0039
PROB0040
PROB 41
PROB 142
PROB 143
PROB 144
PROB 145

DO 5112 J=2, MROW
OUTLIM=PCISON(AV(J), TIME)
REC=DISC(OUTLIM, PROB(J))
5109 K=J-1
5004 IF (REC - OUTLIM) 5008, 5005, 5005
5005 IF (OUTLIM - QUITA(J)) 5007, 5007, 5010
5007 P(J) = OUTLIM
GO TO 5011

5008 IF (REC - QUITA(J)) 5009, 5009, 5010
5009 B(J)=REC
GO TO 5011

5010 B(J)=QUITA(J)
5011 CONTINUE

PERCT(J)=B(J)/QUITA(J)*100.
WRITE OUTPUT TAPE 3,7778,K,B(J), PERCT(J)
7778 FCRRMAT(26X, I2, 11X, F13.0, 15X, F13.3)

5112 CONTINUE
5013 I=2
SUM=0.0
SUM=SUM+B(I)
5014 IF (I-(MROW-1)) 5015, 5016, 5016
5015 I=I+1
GO TO 5014

5016 HOLD = INTF(SUM * PROP)
IF ((HOLD - (SUM * PROP)) - 0.5) 5026, 5027, 5027

5026 B(MROW) = HOLD
GO TO 5028

5027 B(MROW) = HOLD + 1.


```

SUBROUTINE AVERAG(MIKE)
DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50),Y(50),
1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
2KOUT(10),ERR(10),RUN(8),QUOTA(50),PROB(50),AV(50),ANS(50),
3BSTORE(50,52),CLDSS(50),ZBAR(50),PERCT(50)
OCOMMON TCL,R,Y,E,Z,DDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
1INCLL,MROW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
2DDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLDSS,VARCST
EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
AMIKE=MIKE
ANS(MIKE)=Y(1)
SUMANS=SUMANS+ANS(MIKE)
VARCST=VARCST+ANS(MIKE)*ANS(MIKE)
IF (MIKE-1) 5066,5066,5065
5065 DEVCST=(VARCST-AMIKE*AVCST)/(AMIKE-1.)
DEVCST=SCRTF(DEVCST)
GO TO 5067
5066 DEVCST=0.0
5067 PRINT 5057,AVCST,DEVCST
5057 FORMAT(//34X,12H AVERAGE COST,14X,21H STD DEVIATION OF COST//32X,
1F13.2,16X,F13.2//)
RETURN
END

```

PROB 710

C

```

SUBROUTINE RISK
DIMENSION A(50,100),B(50),KB(100),X(50),JH(50),P(50),Y(50),
1E(50,50),Z(100),DDT(100),PINV(50,50),XI(50),INFIX(10),TOL(10),
2KOUT(10),ERR(10),RUN(8),QUOTA(50),PROB(50),AV(50),ANS(50),
3BSTORE(50,52),CLDSS(50),ZBAR(50),PERCT(50)
OCOMMON TCL,R,Y,E,Z,DDT,PINV,XI,A,B,KB,X,JH,P,INFIX,KCUT,ERR,RUN,
1INCLL,MROW,MROW1,MAX,TIME,PROP,SUMANS,BSTORE,AMAX,MAX1,MAX2,PRM,
2DDT,NCOL,MROW,PERCT,QUOTA,PROB,AV,ANS,BSTORE,CLDSS,VARCST
EQUIVALENCE (INFIX(2),NCOL),(INFIX(4),MROW),(NRUN,RUN(1))
VARSK=0.0
TLOSS=0.0
DO 5052 J=2,MROW
BSTORE(J,MAX1)=BSTORE(J,MAX1)/AMAX
5052 WRITE OUTPUT TAPE 3,5070
5070 FORMAT(1F1,///,33X,14H STATION NUMBER ,11X,2CHMEAN NUMBER ENLI
1STED,///)
DO 5071 J=2,MROW
K=J-1
WRITE OUTPUT TAPE 3,5071,K,B(J)
5071 FORMAT(39X12,14X,F13.0)
CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E,DDT)
K=MROW
DO 1001 I=1,MROW

```

LP101010
LP101020

LP101030
LP101040
LP101050
LP101060

```

L=0
DO 1001 J=1,K,MROW
L=L+1
1001 PINV(I,L)=E(J)
39 WRITE OUTPUT TAPE 3,39,Y(1)
FORMAT(1H1,29X,48H MINIMUM CCST OF ALL TRANSPORTATION OF MEANS IS
F13.2)
24 3X,23HRCUTES USED AND AMOUNTS ///36X,7HRCUTES ,16X,18HNUMBER MEN
3 SHIPPED ///
WRITE OUTPUT TAPE 3,5061,(JH(I),X(I),I=2,MROW)
5061 FORMAT(35X,12,19X,F13.0)
5062 ZBAR(M)=C.0
WRITE OUTPUT TAPE 3,5086
5086 FORMAT(1H1,///30X,6HSAMPLE ,5X,23HCOST USING RANDOM INPUT ,5X,
119HLOSS CN THIS SAMPLE ///)
DO 5088 M=1,MAX
5088 DO 5088 K=2,MROW
B(K)=BSCORE(K,M)
DO 5051 I=2,MROW
X(I)=C.0
DO 5051 K=2,MROW
X(I)=X(I)+PINV(I,K)*B(K)
DO 5050 I=2,MROW
IF (X(I)) 5053,5050,5050
5053 IF (XMODF(JH(I),2)) 5054,5054,5056
5054 DO 5055 L=2,MROW
IF (JH(I)-JH(L)-1) 5055,5063,5055
5055 CONTINUE
PRINT 5050
5090 FORMAT(7FERROR 1)
GO TO 5050
5056 DO 5065 L=2,MROW
IF (JH(L)-JH(I)-1) 5065,5063,5065
5065 CONTINUE
PRINT 5095
5095 FORMAT(7FERROR 2)
GO TO 5050
5063 X(L)=X(L)+X(I)
X(I)=C.0
PRINT 5060,M
5060 FORMAT(/10X,29HINFEASIBLE SOLUTION ON SAMPLE ,I2//)
5050 CONTINUE
DO 5064 I=2,MROW
K=JH(I)
ZBAR(M)=ZBAR(M)+X(I)*A(1,K)
CONTINUE
CLOSS(M)=ZBAR(M)-ANS(M)
5064

```



```

TLOSS=TLCSS+CLOSS(M)
VARSK=VARSK+CLOSS(M)*CLOSS(M)
WRITE OUTPUT TAPE 3,5058,M,ZBAR(M),CLOSS(M)
5058 FORMAT(31X,I3,10X,F13.2,12X,F13.2)
AVLOSS=TLOSS/AMAX
VARSK=(VARSK-AMAX*AVLOSS*AVLOSS)/(AMAX-1.)
CEVRSK=SCRTF(VARSK)
WRITE OUTPUT TAPE 3,5059,AVLOSS,DEVRSK
5059 FORMAT(//30X,4HRISK
1, F13.2 //)
RETURN
END

```

C

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